Technical Note

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LEVEL II

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TIME DELAY ESTIMATION: FOR KNOWN & UNKNOWN SIGNALS

by

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I. Introduction

Various optimum and sub-optimum filters have been added to the basic generlized cross-correlator in order to enhance its detection and estimation capabilities in the present of noise. An experimental comparision of the optimum and sub-optimum filters was discussed in Hassab and Boucher[1]. For the random signals, the optimum window $W_{II}(w)$ which they derived has the results of time-delay estimation considerably.

Basically, the solution consists of cross-correlating the sensor outputs. The time argument that corresponds to the maximum peak in the output is the time delay(Fig. 1). In this report, time delay estimation under several assumptions about signals is obtained by averaging the periodgrams of the data segments.[3] When the signal is unknown, the maximum entropy power spectrum estimation of Burg's algorithm is used to estimate the signal power spectrum from the measurement of separated sensors. The Akaike FPE(Final power error)criterion is also concerned in deciding the order of AR model and signal & noise power spectrum estimation of Burg's algorithm.

II. Theory

Let the sensor output be the form

$$\begin{aligned} Z_1(t) &= y(t)+n_1(t) \\ Z_2(t) &= ay(t-\tau)+n_2(t) \end{aligned} \tag{1}$$

where the signal $y(t)$ and noises $n_1(t)$, $n_2(t)$ are uncorrelated and jointly stationary zero mean Gaussian random processes over an observation interval $T \gg \tau$. Here τ is the difference in arrival time or the time delay between the two sensor elements.

In the correlator, the multiplier output is given by[2]

$$Z_1^*(w)Z_2(w) = a\,\phi_y(w)\,e^{-jw} + \phi_{n1n2}(w) + \phi_c(w) \tag{2}$$

where

where

$$\phi_y(w) = |Y(w)|^2 \qquad , \ \phi_{n_1n_2}(w) = N_1^*(w)\ N_2(w)$$

$$\phi_c(w) = Y^*(w)N_2(w) + aY(w)e^{-jw\tau}N_1^*(w)$$

The lower case ϕ refers to a single realization of the function and the upper case indicates its expected value over the ensemble, i.e. $<\phi_y(w)> = <|Y(w)|^2> = \Phi_y(w)$. The window $W_{II}(w)$ derived by Hassab and Boucher [2] is based on the optimal detection criterion of maximizing the ratio of mean correlator output due to signal and noise present. The window function can be expressed in the simple form as

$$W_{II}(w) = \frac{\Phi_y(w)}{\Phi_{z1}(w)\Phi_{z2}(w)} \qquad (3a)$$

or

$$W_{II}(w) = \frac{\Phi_y(w)}{\Phi_{n1}(w)\Phi_{n2}(w) + \Phi_y(w)\left[\Phi_{n2}(w) + a^2\,\Phi_{n1}(w)\right] + a^2\Phi_y(w)} \qquad (3b)$$

With a finite number of samples, one spectral density estimate is the periodogram. A good spectral estimate is obtained by averaging several periodogram from segments, denoted as M, that provides the best spectral density estimate and the time-delay estimate.

For deterministic signals, the resulting optimum filters are in terms of signal and noise spectral densities.

$$W_I(w) = \frac{\Phi_y(w)}{\Phi_{n1}(w)\Phi_{n2}(w) + \Phi_y(w)(\Phi_{n2}(w) + a^2\Phi_{n1}(w))} \qquad (4a)$$

or

$$W_I(w) = \frac{\Phi_y(w)}{\Phi_{z1}(w)\Phi_{z2}(w) - a^2\Phi_y^2(w)} \qquad (4b)$$

the difference between W_I and W_{II} is due to the variance $a^2\Phi_y^2$ of the signal which is negligible for small a and/or low signal-to-noise spectra.

Otherwise, both filters are similar where the first term within brackets in the denominator is due to $\phi_{n_1n_2}(w)$ and the second term $\phi_c(w)$. Hassab and Boucher[4] have examined the behavior of every window in the presence of a strong spectral peak in $\phi_y(w)$. With a no noise case $W_I(w)$ is undefined and $W_{II}(w)$ is reduced to $\frac{1}{\Phi_y(w)}$. The expected generlized correlator output becomes

$$Z_1(w)Z_2(w) = a\Phi_y(w)e^{-jw\tau}$$

divided by $\Phi_y(w)$(since the window $W_{II}(w)=\frac{1}{\Phi_y(w)}$) then, we get the term of $e^{-jw\tau}$, the time delay estimation. Hence in this case we use $W_{II}(w)$ to process the signal of sinusoid ...

In the case of signal unknown Burg's method is used to estimate the $\Phi_y(w)$, $\Phi_{z1}(w)$ and $\Phi_{z2}(w)$.

MAXIMUM ENTROPY SPECTRUM ESTIMATION: Burg's Algorithm

The maximum entropy method(MEM) using the Burg's algorithm was applied by Ulrych[5],who showed the remarable resolution properties of this approach. In applying the concept of maximum entropy to spectral analysis [6] we begin with the relationship between the entropy(strictly speaking, the entropy rate for an infinite process) and the spectral density S(f) of a stationary Gaussian process,

$$H = \frac{1}{4f_N}\int_{-f_N}^{f_N} \log S(f)\,df \qquad (5)$$

where f_N is the Nyquist frequency.

Rewritting (5) in terms of the autocorrelation $\phi(k)$ of the process gives

$$H = \frac{1}{4f_N}\int_{-f_N}^{f_N} \log\left[\sum_{-\infty}^{\infty} \phi(k)\exp(-2j\pi fk\Delta t)\right]df \qquad (6)$$

where Δt is the uniform sampling rate. Maximizing (6) with respect to the unknown $\phi(k)$ with the constraint that S(f) must also be consistent with the known autocorrelations $\phi(0),\phi(1),\ldots,\phi(m-1)$ results in the MEM spectral estimate.

This estimate expresses maximum uncertainty with respect to the unknown information but is consistent with the known information. The variational procedure

The estimate expresses maximum uncertainty with respect to the unknown information but is consistent with the known information. The variational procedure leads to the same expression for the MEM spectral density [6], [7], which for a real linear process x(t) is

$$P_E(f) = \frac{P_M}{f_N \left| 1 + \sum_{i=1}^{M-1} r_i \exp(-j2\pi f i \Delta t) \right|^2} \qquad (7)$$

where P_M is a constant and r_i are prediction error coefficients that are determined from the data.

The chief shortcoming of the MEM spectral estimates has been the lack of a quantitative method of determing the length of the prediction error filter r(t) in (7). Recent work of AR processes appears to overcome this problem.

The algorithm of Burg's method is considered a wide-sense stationary Gaussian process x(t) of zero mean and duration T seconds. The detail of this algorithm is in reference [A].

III. Experimental Study

(A) Signal known:

The random signal is generated by passing white noise through a recursive filter with the impulse response Bnexp(-Bn). The Gaussian white noise at the outputs of two sensors are independent and have been generated by Monte Carlo's method. An implementation of the generlized correlator is executed for each set of 128 data points obtained from the sensor outputs. The time delay between the sensor elements is arbitrary and is selected as 16 & 10 units with different case. Here to compare the time delay results from different signal-to-noise ratios and to determine the optimum M.

When the signal y(t) is random with known statistics plus the Gaussian noise with zero mean and variance , the signal-to-noise ratios of -10dB, 0dB, and 10dB are chosen with results given in Figs. 2-6 with B=3.33, τ=16. Figs. 7-11 with B=17, τ=10.

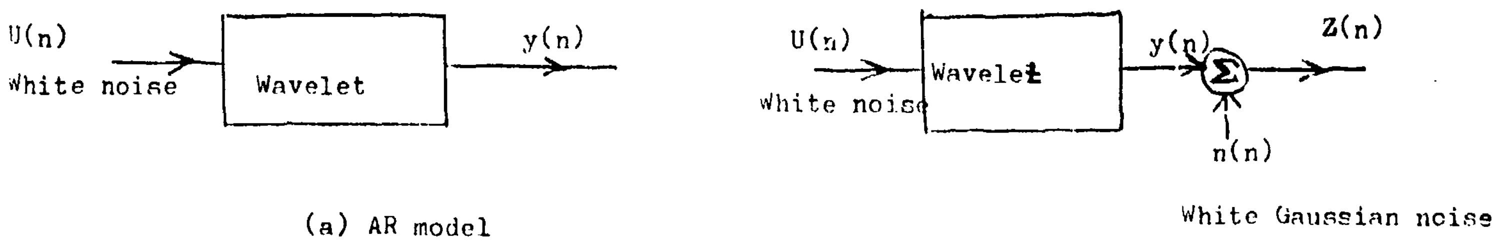
The sinusoidal signal y(t) is known and plus the Gaussian noise, we have the results shown in Figs. 12-14.

(B) Signal unknown:

From the parameter estimator[8] which with the addition of noise in the output of the AR model.is

$$Z(n)=y(n)+n(n)$$

Hence, the model for the sinusoids in white noise problem contains poles and zeros. In using AR parameter estimation methods that assume all-pole model is difficulty.[9][10] It is well known that the least squares estimateof coefficient is biased in this case, and the bias is caused by the expected value of $E[Z^T\epsilon] \neq 0$ ($W=aZ+\epsilon$). Corresponding to the FPE we can get $\Phi_y(w)$ by substracting $\Phi_z(w)$ with FPE.



(a) AR model

(b) ARMA model

Block diagram (a) AR model (b) ARMA Model

Then, use this power spectrum as the window parameter of Hassab & Boucher optimum window, and we can estimate the time delay.from a generlized correlator.

Similar to the known signal case when y(t) is random with unknown statistics plus the Gaussian noise with zero mean and variance, the signal-to-noise ratios of -10dB,0dB,and 10dB are chosen with results given in Figs. 15-19 with B=3.33,τ=16. In Figs 20 24 with B=17 ,τ=10.

The sinusoidal signal y(t) is unknown and plus the unknown white Gaussian noise, we have the results shown in Figs. 25-27 ,with τ=16.

IV. Summary

The time delay estimation from cross-correlator without window exhibits a thresholding effect as the signal-to-noise ratio decreases. In which we get the false peak.[11]

After using the window and averaging the periodgrams of the data segments the thresholding effect can be avoided then the maximum M is chose. The unknown signal case before we compute the $\Phi_y(w)$ the Akaike FPE criterion must be used to decide the order of the prediction error filter coefficients.(See Table 1.)

The relation of SNR and M can be found in Fig.28 from the Fig. we found M is increased when SNR is decreased.

In the random signal case B=3.33 and B=17 are chosen. Here we can find B=17 has the small power compared to B=3.33 and the small M(=7) is enough to detect the time-delay with SNR=-30dB.

The key of time delay estimation of unknown signal is the estimation of $\Phi_y(w)$, $\Phi_{z1}(w)$, and $\Phi_{z2}(w)$. If we can estimate the power spectrum of y(t) from the sensor outputs, then we must have a good estimate using the above method.

References

1. J. C. Hassab and R.E. Boucher, " An experimental comparison of optimum and sub-optimum filters effectiveness in the generlized correlator," Journal of Sound and Vibration, vol. 76, pp. 117-128.,1981.

2. J.C. Hassab and R.E. Boucher, " Optimum estimation of time delay by a generlized correlator," IEEE Trans. on Acoustics, Speech and Signal Processing, vol. ASSP-27, pp. 373-380, August 1979.

3. A. Oppenheim and R.W. Schafer, "Digital Signal Processing," Prentice-Hall, 1975.

4. J. C. Hassab and R.E. Boucher, "Performance of the Generalized Cross-correlator in the presence of a strong spectral peak in the signal," Naval Underwater Systems Center, Newport,R.I.

5. Ulrych,T. J., " Maximum entropy power spectrum of truncated sinusoids," J. Geophys. Res., 77, 1396-1400, 1972 .

6. Smylie, D.,E., G. K. C. Clarke, and T. J. Ulrych, " Analysis of irregularities in the earth's rotation, in Methods in Computational Physic, vol. 13,pp. 391-430,Academic, New York, 1973.

7. Edward, J. A., and M. M. Fitelson,"Notes on maximum-entropy processing, " IEEE Trans. Theory, IT-19,232,1973.

8. Y. T. Chan,and J.M.M.Lavoie, and J.B. Plant," A parameter estimation approach to estimation of frequencies of Sinusoids," IEEE Trans. ASSP.,vol. 29,No.2, April 1981.

9. Y.W. Chen and G.R. Stegen, " Experiments with maximum entropy power spectra of sinusoids," J. Geophys. Res., vol. 79, July 1974.

10. S. Kay," Maximum entropy spectral estimation using the analytical signal," IEEE Trans. ASSP, vol. 26,Oct. 1978.

11. John p. Ianniello, " Threshold effects in Time-Dealy estimation via cross-correlation," Naval Underwater Systems Center, New London Laboratory—Code 3212.

12. S. Haykin," Communication systems," p476, John Wiley and Sons,1978

13. J.P. Burg, " Maximum entropy spectral analysis," paper presented at 37th meeting,sec. of Explor. Geophys. Oklahoma city,Okla. Oct. 1967.

Figure Captions

Fig. 1 Block diagram of the generalized cross-correlator.
Fig. 2a Without window output for M=1, =16, SNR=10dB, the result of time-delay =16.
Fig. 2b window function of $W_{II}(w)$, for M=1, τ=16, SNR=10db.
Fig. 2c With window of $W_{II}(w)$, M=1, SNR=10dB, =16, the result of time-delay =16.
Fig. 3a Without window output for τ=16, SNR=0dB, M=1, the result of time-delay =10.
Fig. 3b Window function of $W_{II}(w)$, for M=1, τ=16, SNR=0dB.
Fig. 3c With window of $W_{II}(w)$, for M=1, τ=16, SNR=0dB, the result of time-delay =13.
Fig. 4a The same as Fig. 3c, but M=3, the result of time-delay =13.
Fig. 4b The same as Fig. 3c, but M=7, the result of time-delay =16.
Fig. 5a Without window output for =16, M=1, SNR=-10dB, the result of time-delay =-6.
Fig. 5b Window function of $W_{II}(w)$, for M=1, τ=16, SNR=-10dB.
Fig. 5c With window of $W_{II}(w)$, M=1, SNR=-10dB, τ=16, the result of τ=13.
Fig. 6a With window output for M=7, SNR=-10dB, the result of time-delay =12.
Fig. 6b With window for M=15, SNR=-10dB, the result of time-delay =13.
Fig. 6c With window for M=20, SNR=-10dB, the result of time-delay =14.
Fig. 7a Without window B=17. τ=10, SNR=10dB, M=1, the result of time-delay =10.
Fig. 7b With window B=17., τ=10, SNR=10, M=1, the result of time delay =10.
Fig. 8a Without window, for B=17, SNR=0dB, =10, M=1, the result of time-delay =10.
Fig. 8b With window for B=17., SNR=0dB, τ=10, M=1, the result of time delay =10.
Fig. 9a Without window, for B=17, SNR=-10dB, =10, the result of time-delay =13.
Fig. 9b With window for B=17, SNR=-10dB, =10, M=1, the result of time-delay =10.
Fig. 10a Without window B=17, SNR=-20db, M=1, τ=10, the result of time-delay =13.
Fig. 10b With window B=17, SNR=-20dB, M=1. τ=10, the result of time-delay =13.
Fig. 10c With window B=17, SNR=-20dB, M=5, τ=10, the result of time delay =10.
Fig. 11a Without window B=17, SNR=-30dB, M=7, τ=10, the result of time delay =22.
Fig. 12a Without window sine wave for M=1, SNR=5dB, τ=16, the result of time delay =16.
Fig. 12b With window sine wave for M=1, SNR=5dB, τ=16, the result of time delay =16.
Fig. 13a Without window sine wave for M=1, SNR=-5dB, τ=16, the result of time delay =11..
Fig. 13b With window of sine wave for M=1, SNR=-5dB, τ=16, the result of time delay =17.
Fig. 14a Without window of sine wave for M=1, SNR=-15dB, τ=16, the result of time delay =-20.
Fig. 14b With window of sine wave for M=20, SNR=-15dB, τ=16, the result of time delay =12.
Fig. 15a Without window output signal unknown for B=3.33, M=1, τ=16, SNR=10dB, the time-delay =16.
Fig. 15b Window function of $W_{II}(w)$, for B=3.33, SNR=10db, τ=16.
Fig. 15c With window of $W_{II}(w)$, for B=3.33, SNR=10dB, τ=16, the time delay =16.
Fig. 16a Without window output of B=3.33, SNR=0dB, τ=16, (unknown signal), the result of time delay =13. M=1.
Fig. 16b Window function of $W_{II}(w)$, for B=3.33, SNR=0dB, τ=16, (unknown signal), M=1.
Fig. 16c With window of $W_{II}(w)$ for B=3.33, SNR=0dB, τ=16, (unknown signal), the result of time delay = 13 , M=1.
Fig. 17a Without window output with unknown signal, for B=3.33, SNR=0dB, τ=16, the result of time delay =15, M=3.
Fig. 17b The same as 17.a but M=5, the result of time delay =15.
Fig. 17c The same as 17.a&b, but M=7, the result of time delay =14.
Fig. 18a Without window of unknown signal B=3.33, SNR=-10dB, τ=16, M=1, the result of time delay =-6.
Fig. 18b Window function of $W_{II}(w)$, B=3.33, SNR=-10dB, =16, M=1, unknown signal.
Fig. 18c With window of $W_{II}(w)$, B=3.33, SNR=-10dB, =16, M=1, the result of time delay =11.

Fig. 19a With window ,the same as 18-c, but M=7, the result of time delay τ=11.

Fig. 19b The same as 19-a, but M=15, the result of time delay τ=14.

Fig. 20a Without window of unknown signal for B=17.,SNR=10dB, τ=10, M=1, the result of time delay τ=10.

Fig. 20b With window of unknown signal for B=17,SNR=10dB, τ=10,M=1, the result of time delay τ=10.

Fig. 21a Without window of unknown signal, for B=17, SNR=0dB, τ=10, M=1, the result of time delay τ=10.

Fig. 21b With window of unknown signal, for B=17, SNR=0db, τ=10,M=1, the result of time delay τ=10.

Fig. 22a Without window of unknown signal, for B=17., SNR=-10dB, M=1, the result of time delay τ=13.

Fig. 22b With window of unknown signal, for B=17, SNR=-10dB, τ=10., the result of time delay τ=10, M=3.

Fig. 23a Without window of unknown signal, for B=17, SNR=-20dB, τ=10,M=1, the result of time delay τ=13.

Fig. 23b With window of unknown signal, for B=17, SNR=-20dB, =10,M=1, the result of time delay =10.

Fig. 24a Without window of unknown signal for B=17, SNR=-30dB, τ=10, M=1, the result of time delay τ=-6.

Fig. 24b Without window of unknown signal for B=17, SNR=-30dB, τ=10, M=7, the result of time delay τ=22.

Fig. 24c With window of unknown signal for B=17, SNR=-30dB, τ=10, M=7, the result of time delay τ=12.

Fig. 25a Without window of sine wave of unknown signal, for τ=16,M=1, the result of time delay τ=16. SNR=5dB.

Fig. 25b With window of sine wave of unknown signal, for τ=16, M=1, the result of time delay =16. SNR=5dB.

Fig. 26a With window of unknown signal M=1, the result of time delay τ=15. (without window case is the same as Fig.12a.) (SNR=-5dB).

Fig. 26b With window M=3, the result of time delay τ=16. (SNR=-5dB)

Fig. 27a With window for SNR=-15db, τ=16,M=3, the result of time delay τ=14.

Fig. 27b Without window for SNR=-15dB,M=3, τ±16, the result of time delay τ=12.

Fig. 28 The relationship of random signal case(btexp(-Bt).(known signal)

Note: Fig.0 is the random signal case with B=3.33 and signal is known.

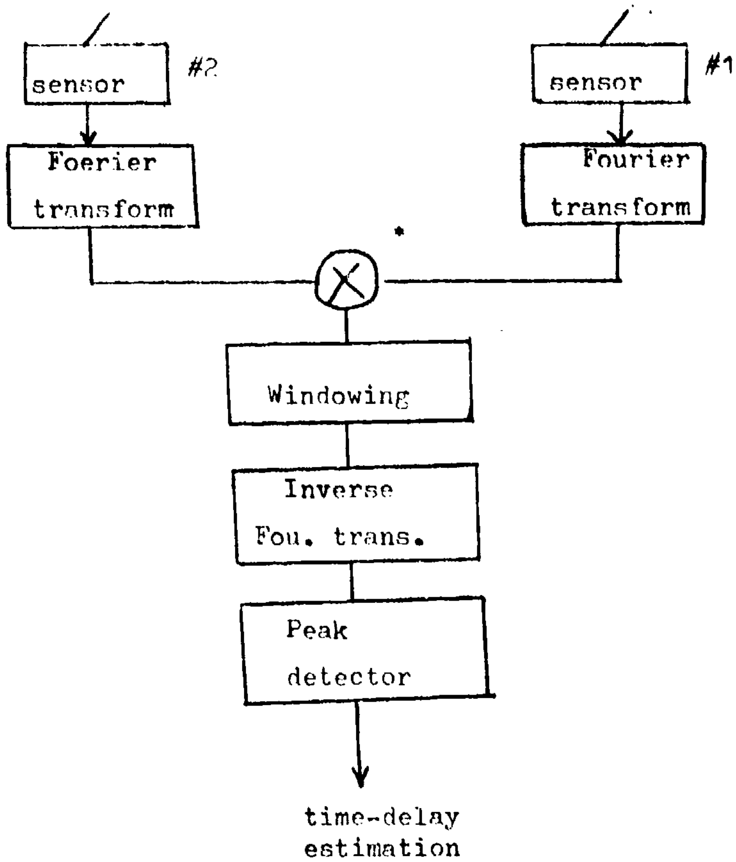
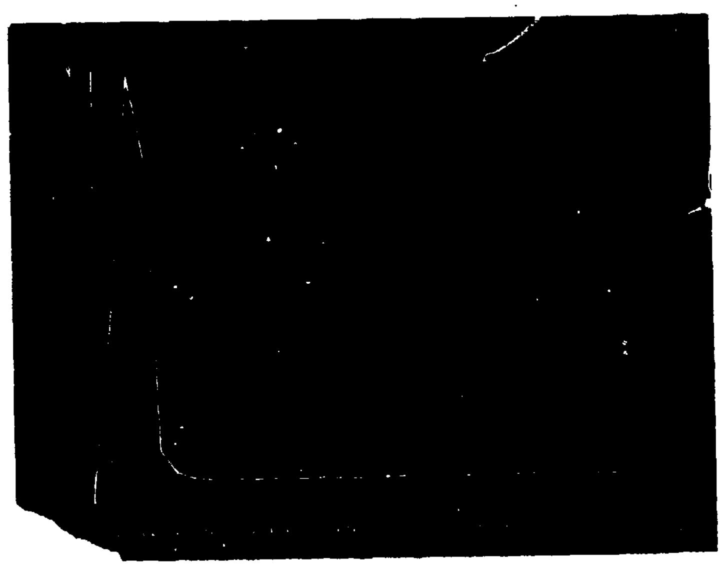

sensor
#2
sensor
#1
Foerier transform
Fourier transform
Windowing
Inverse Fou. trans.
Peak detector
time-delay estimation

Fig. 1

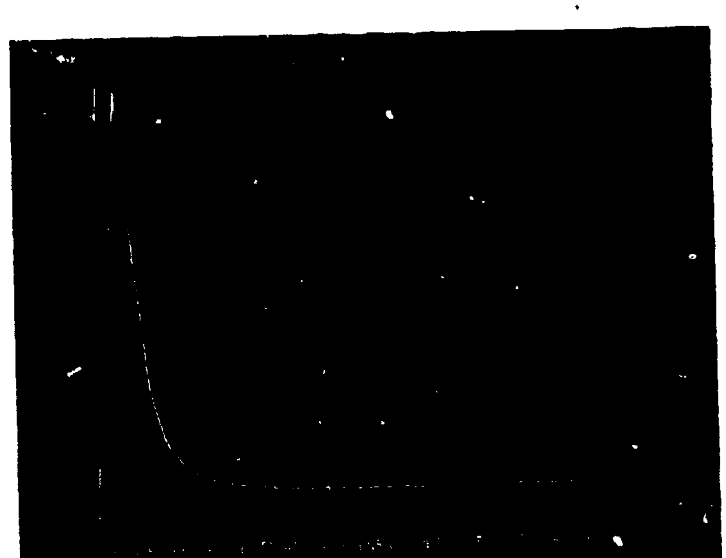
Burg's MEM
(Noiseless.)

Fig.0-a



(10dB)

Fig.0-b



(0dB)

Fig.0-c

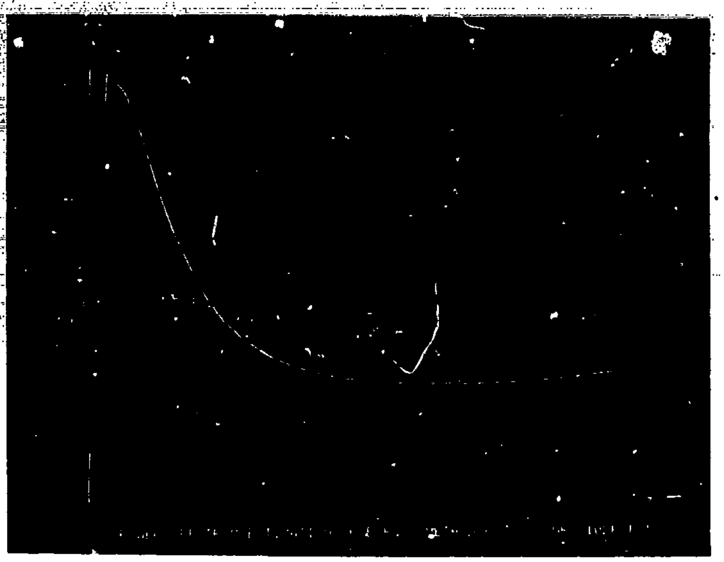


Fig.0-d

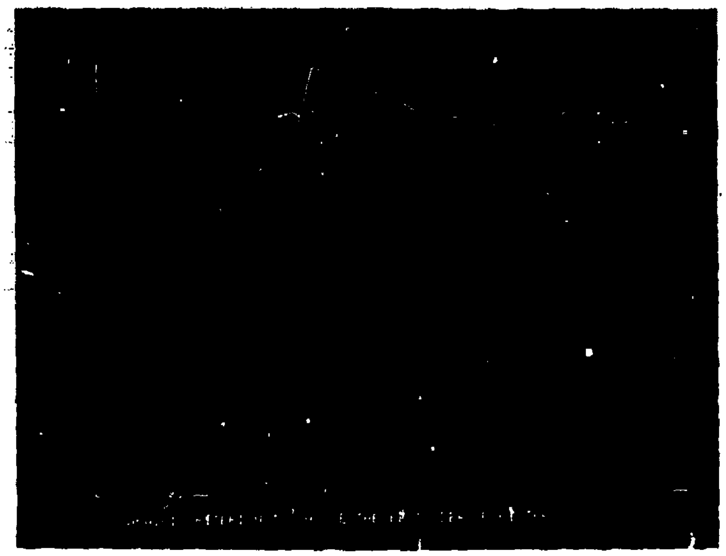
(-10dB)

Fig.0-e

Akaike Criterion

chose the order p

(0dB)



(0dB)

Fig.0-f

(10dB)

Fig.0-g

P	FPE SNR= dB	FPE SNR= dB	FPE SNR= dB
1	0.00000000D 00	0.00000000D 00	0.00000000D 00
2	0.14653313D 00	0.21046814D-01	0.28814746D-02
3	0.14580477D 00	0.18930817D -01	0.25835737D-02
4	0.14234533D 00	0.17784937D -01	0.25794779D-02
5	0.14587360D 00	0.18173976D -01	0.26410812D-02
6	0.14400342D 00	0.18128452D -01	0.27238533D-02
7	0.14859111D 00	0.18672060D -01	0.26400465D-02
8	0.15337423D 00	0.19222392D -01	0.26385433D-02
9	0.15469602D 00	0.19167673D -01	0.25518556D-02
10	0.15856075D 00	0.19745105D -01	0.26324285D-02
11	0.16032657D 00	0.19689272D -01	0.25363746D-02
12	0.16338778D 00	0.20222895D -01	0.26197359D-02
13	0.16487075D 00	0.20723437D -01	0.27046300D-02
14	0.16946066D 00	0.20919714D -01	0.26527577D-02
15	0.17335856D 00	0.21596970D -01	0.27415610D-02

Table 1

The random signal of B=3.33 (y(t)=3t exp(-3t))

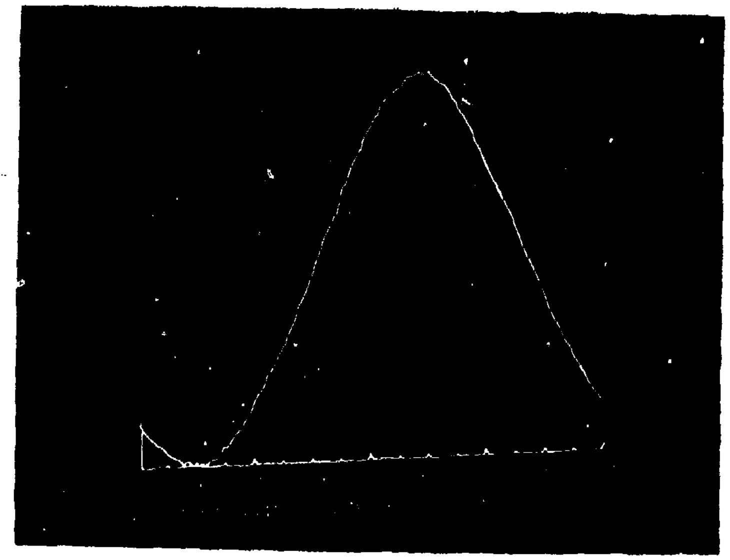


Fig.2-a

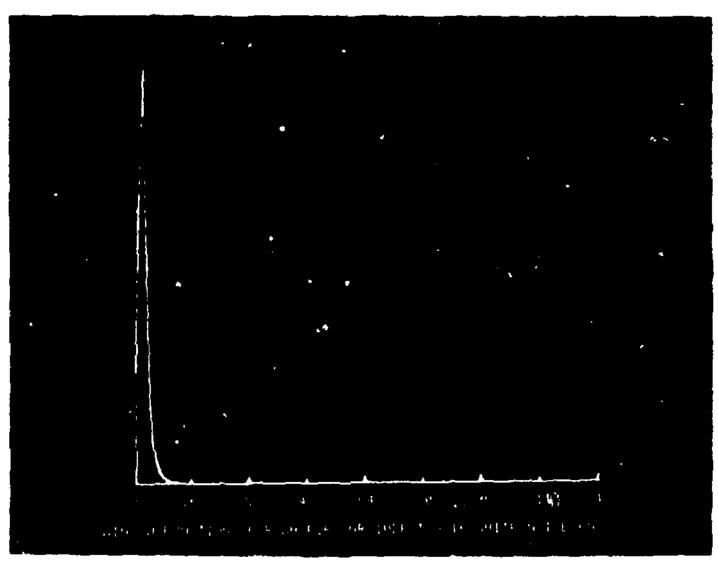


Fig.2-b

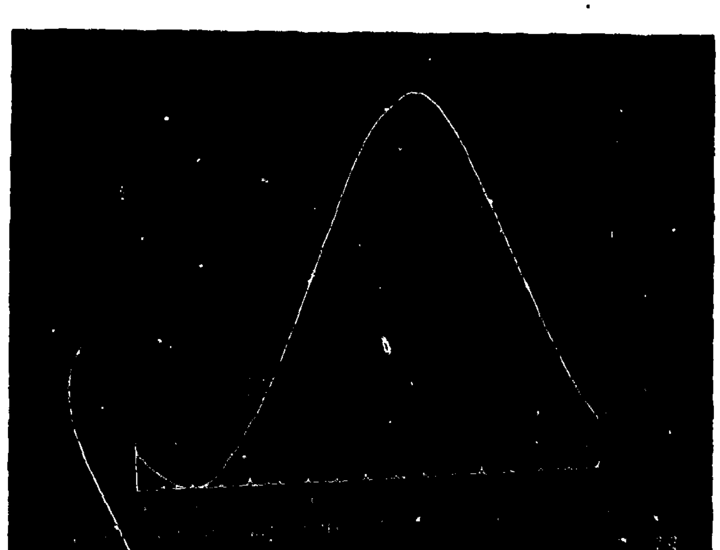


Fig.2-c

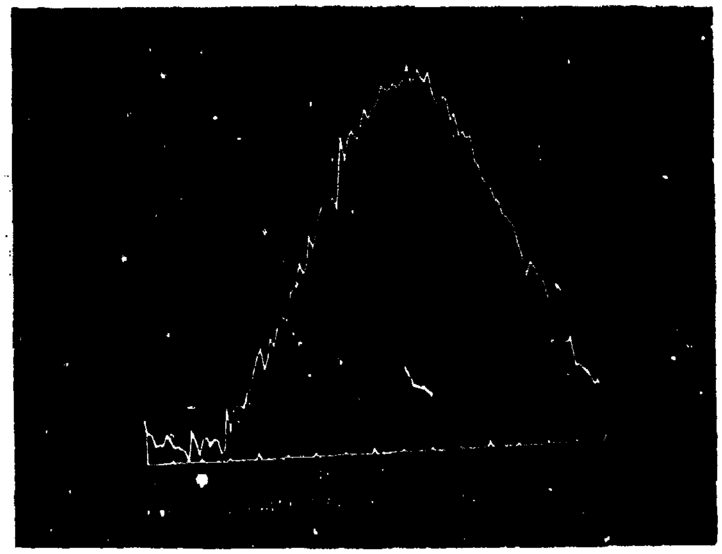


Fig.3-a

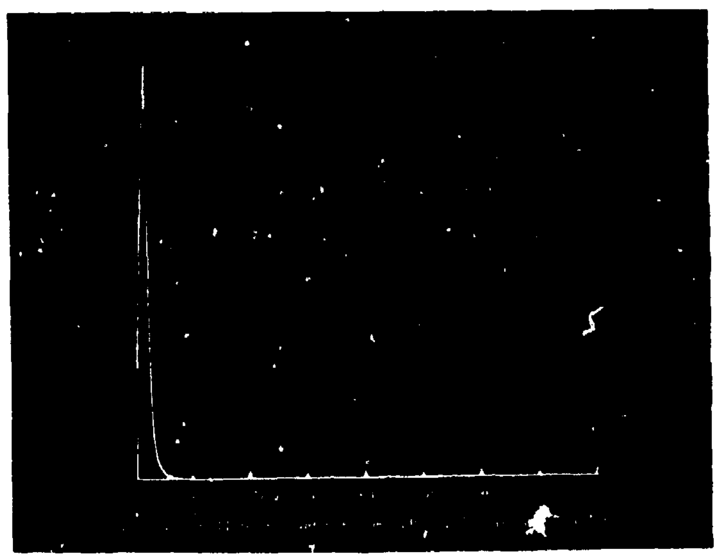


Fig.3-b

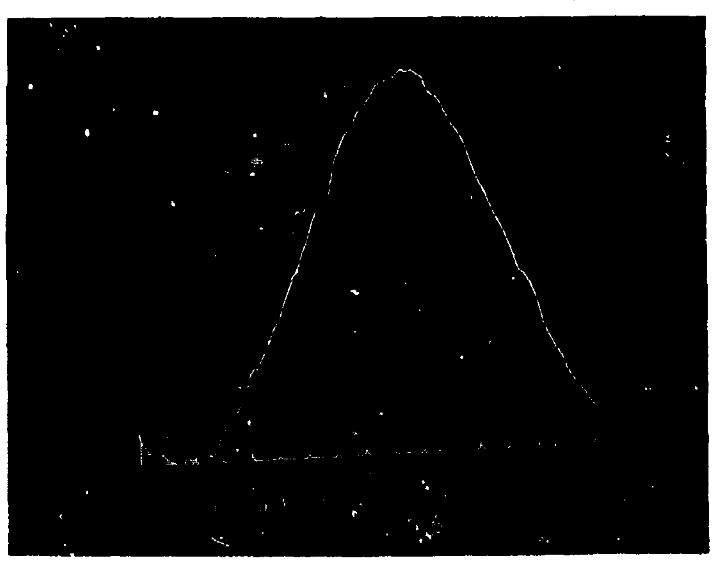


Fig.3-c

Fig.4-a

Fig.4-b

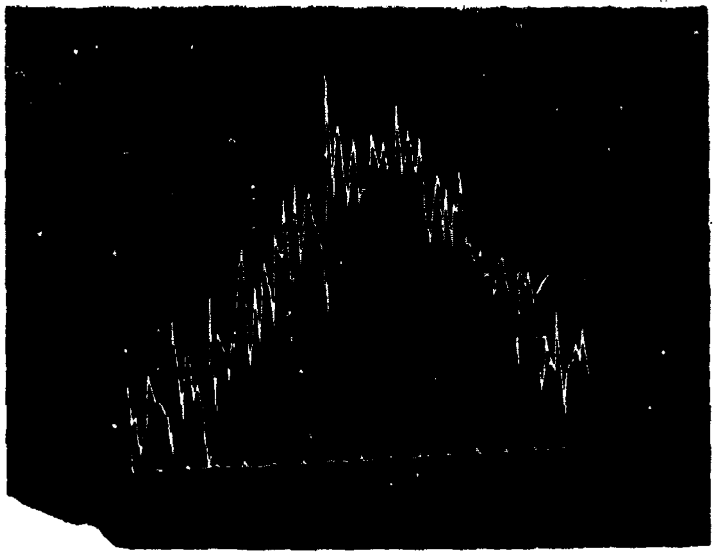


Fig. 5-a

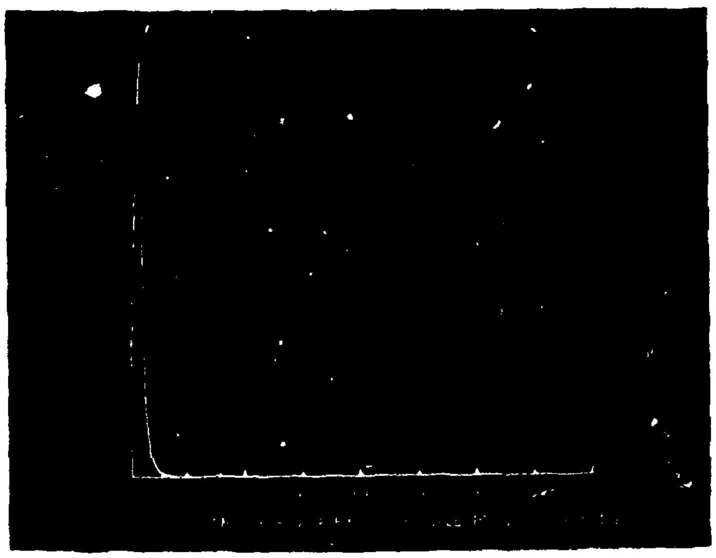


Fig.5-b

Fig.5-c

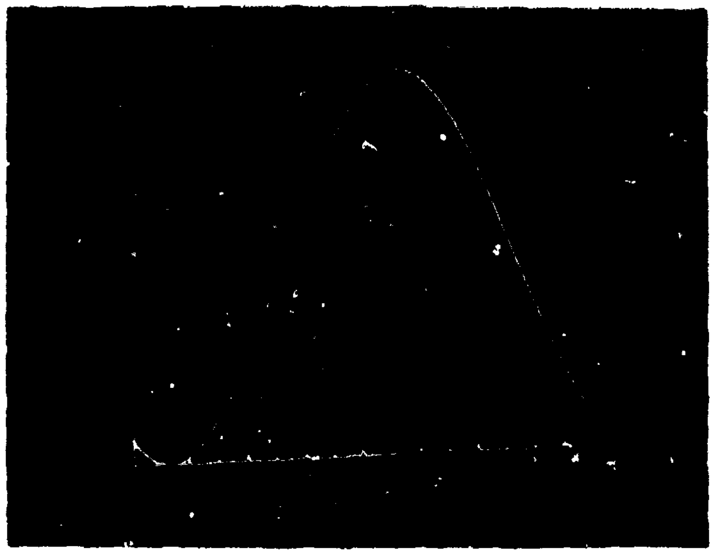


Fig. 6-a

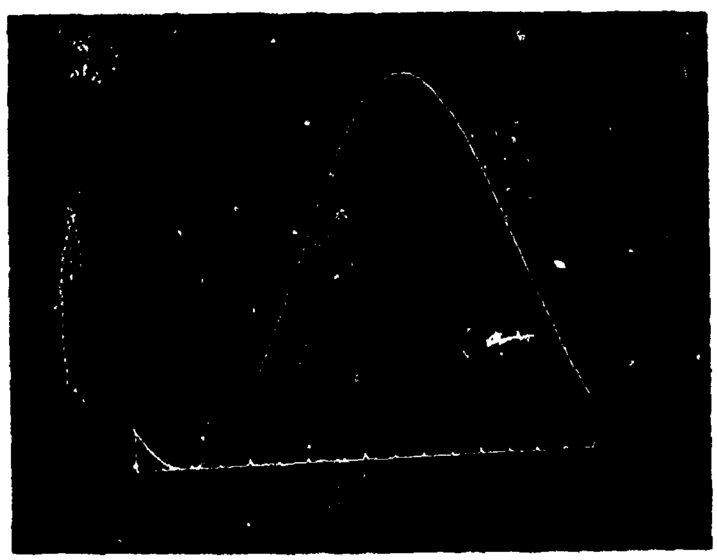


Fig. 6-b

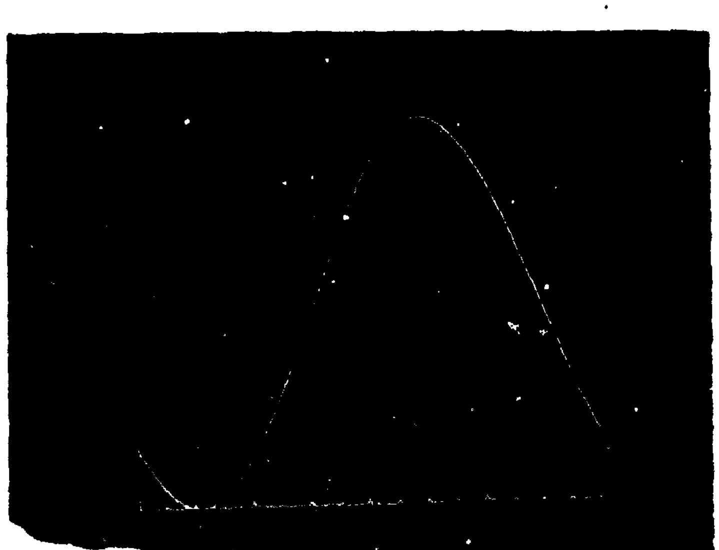


Fig. 6-c

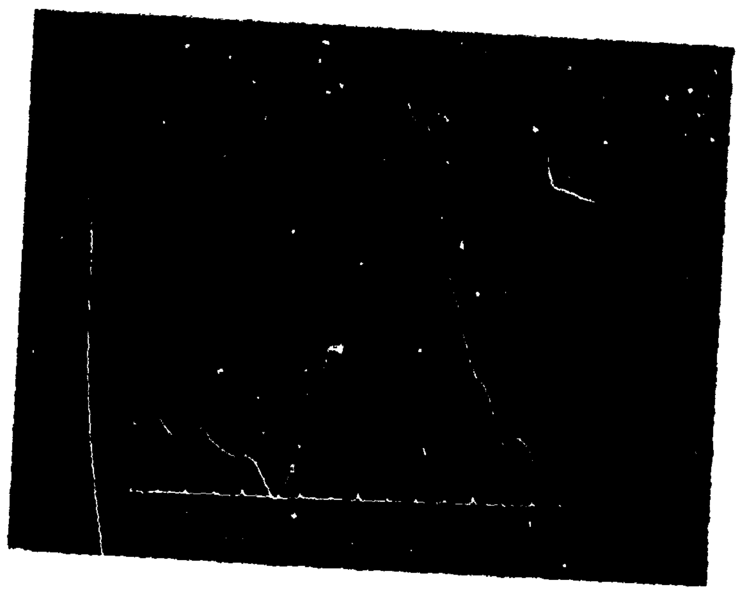


Fig. 7-a

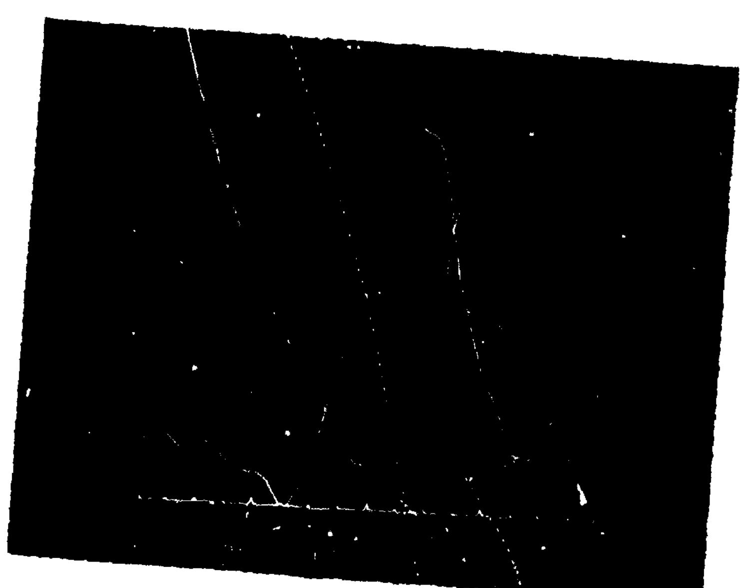
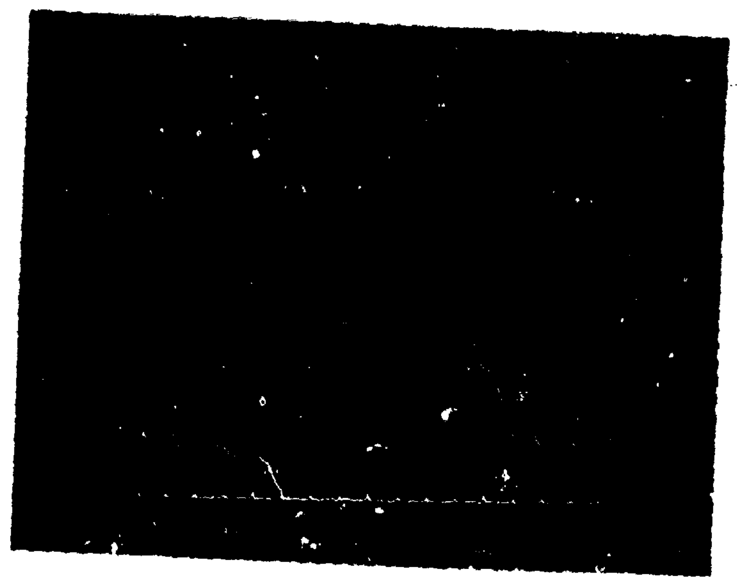


Fig.7-b



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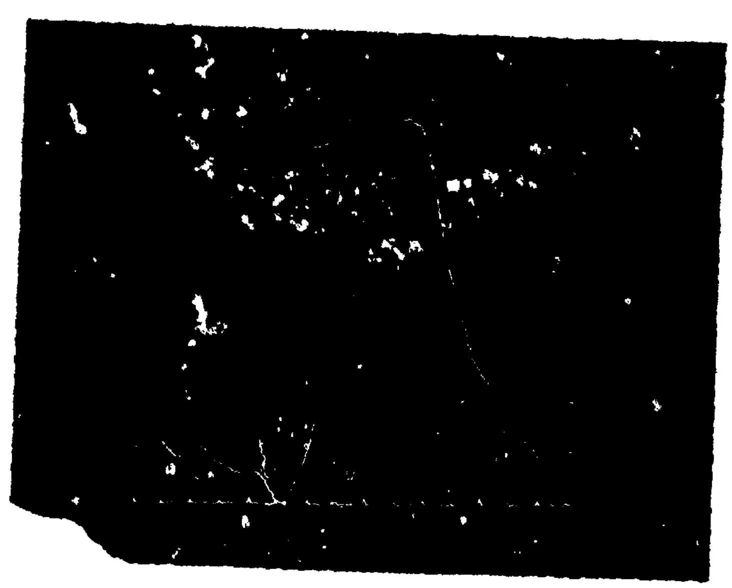


Fig. 8-b

Fig. 9-a

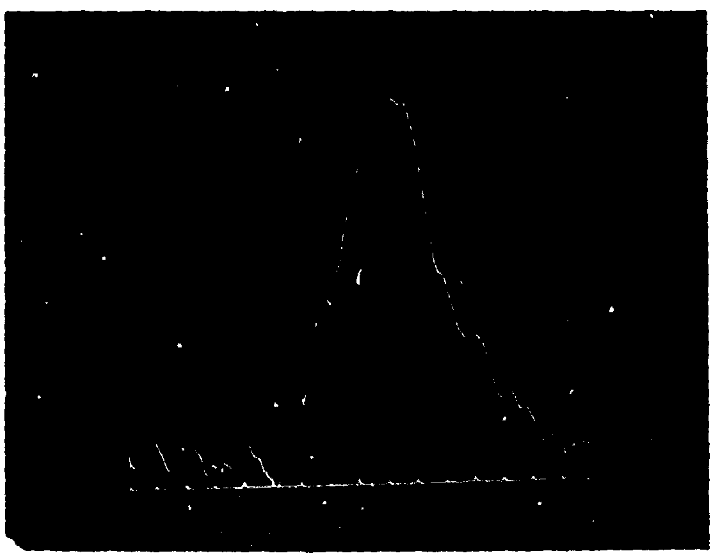
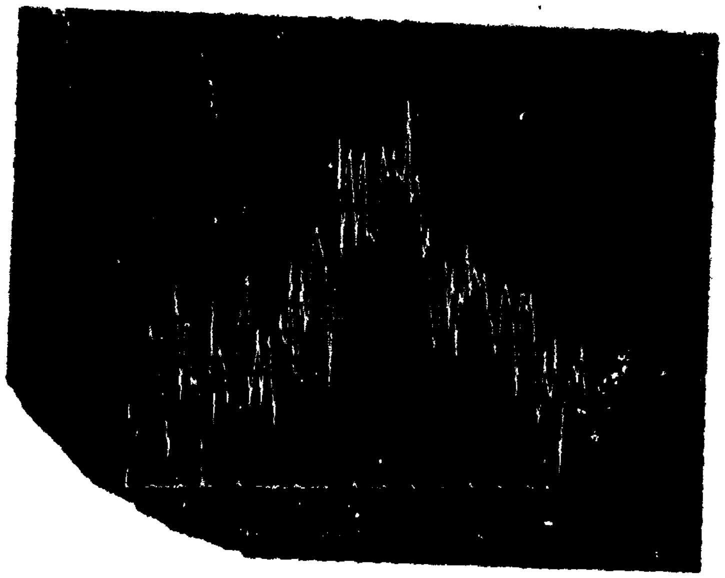


Fig. 9-b



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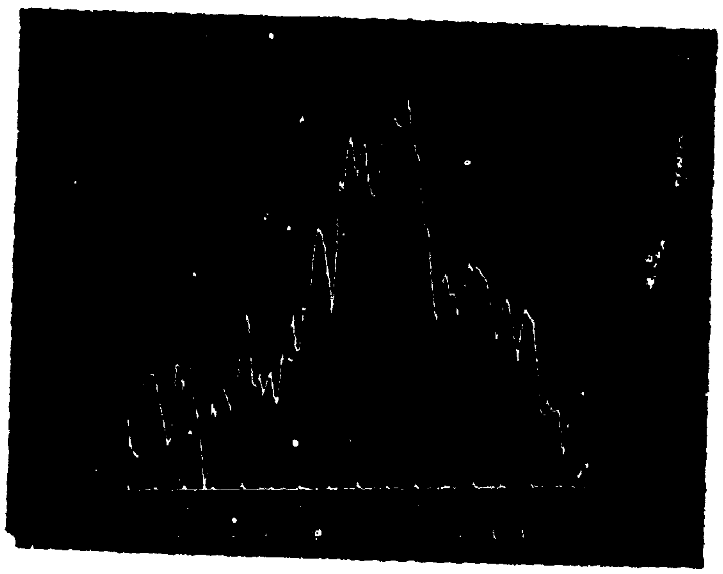


Fig. 10-b

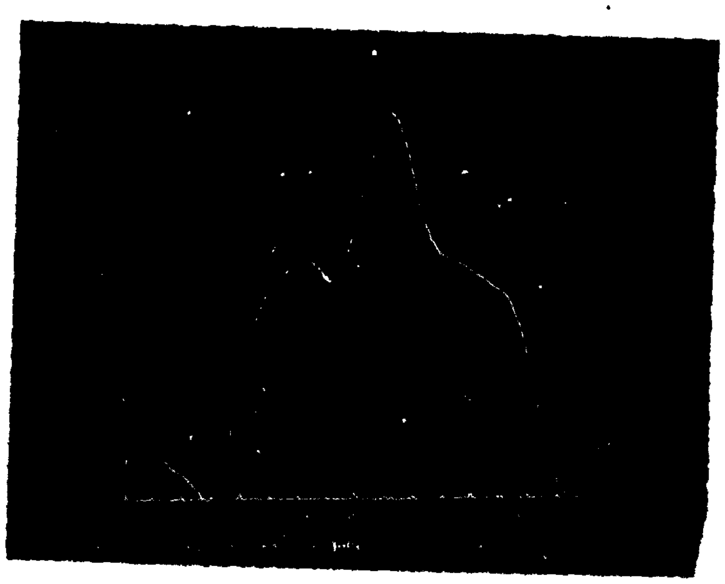
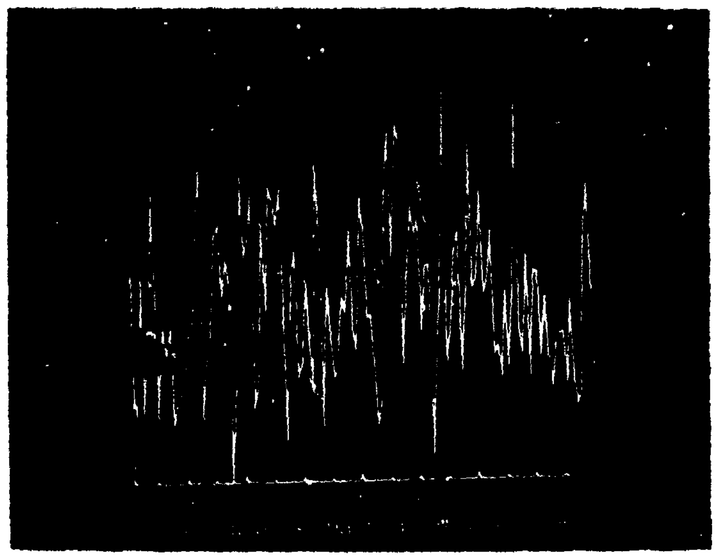


Fig. 10-c



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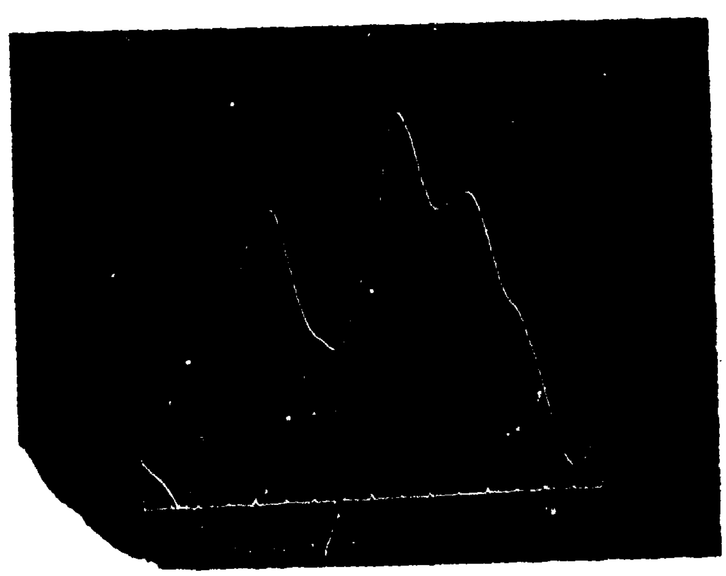
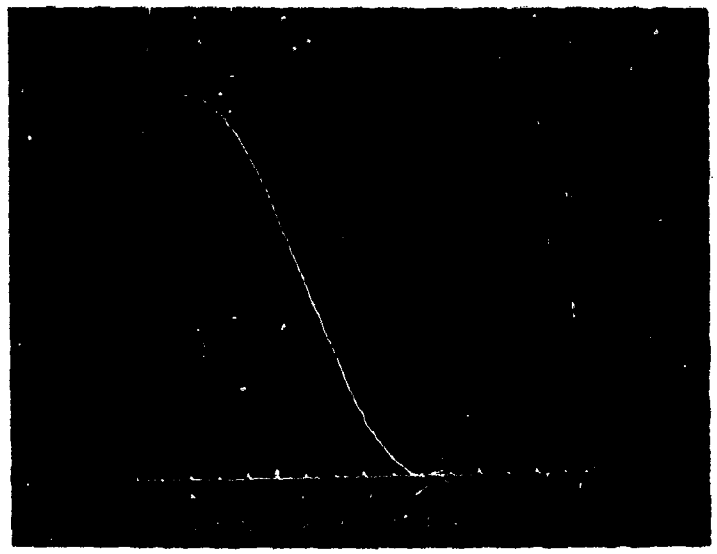


Fig. 11-b



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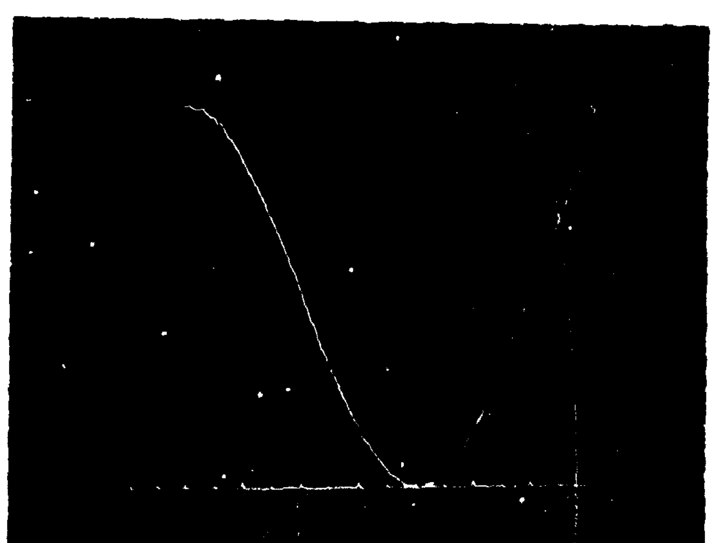


Fig. 12-b

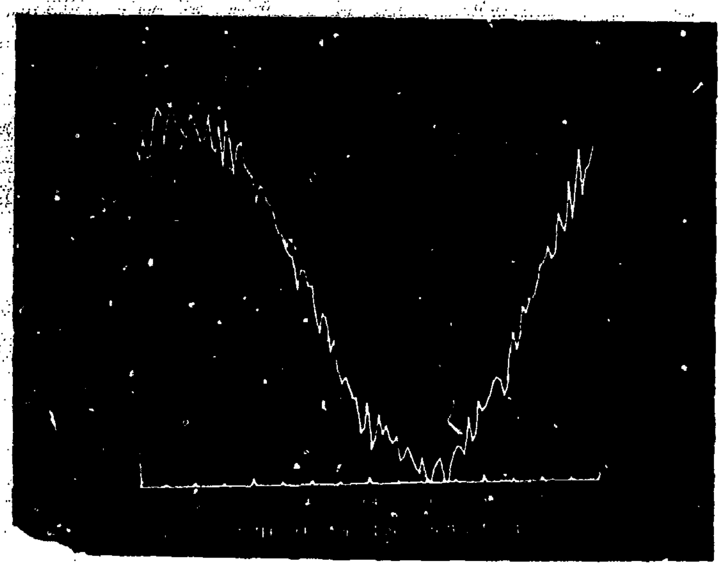


Fig. 13-a

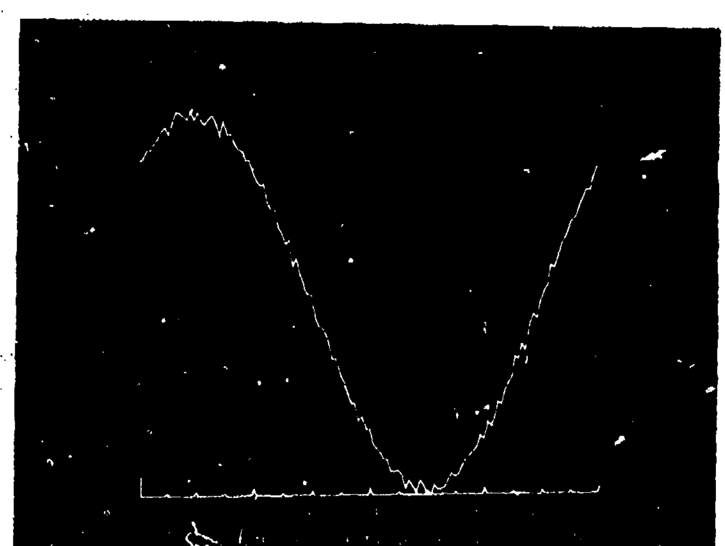


Fig. 13-b

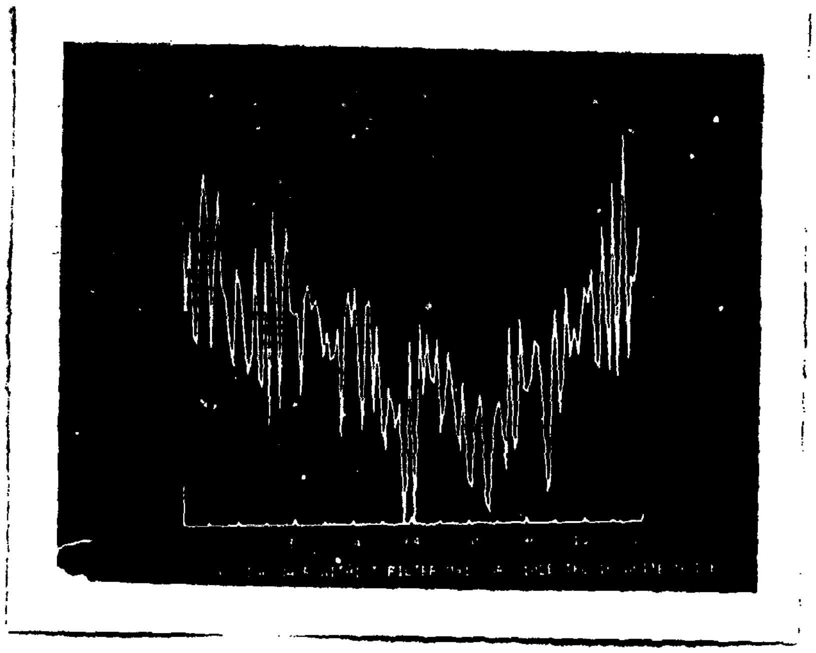


Fig. 14-a

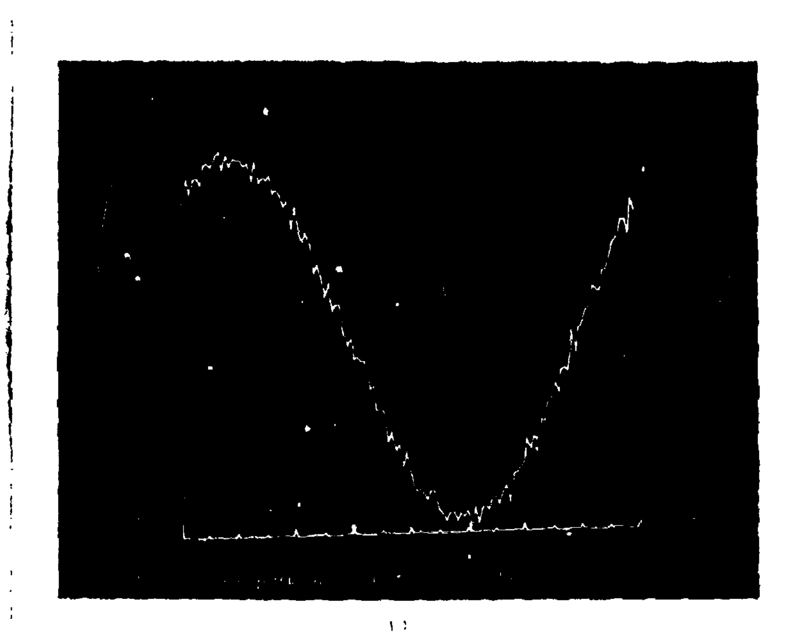


Fig. 14-b

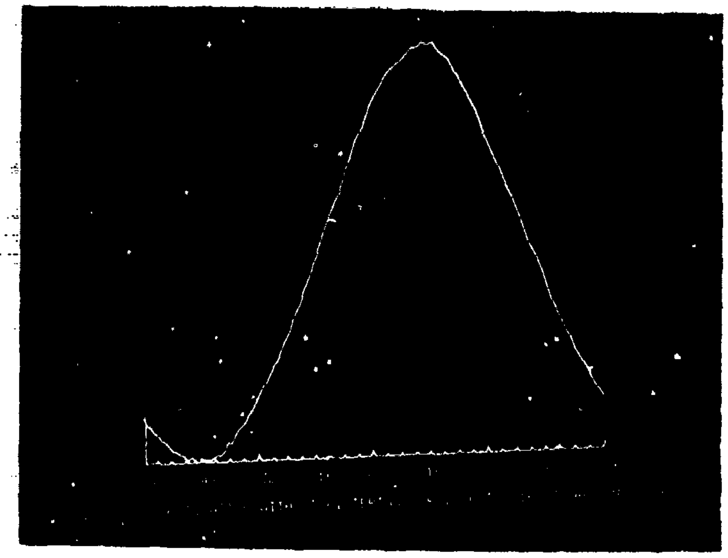


Fig. 15-a

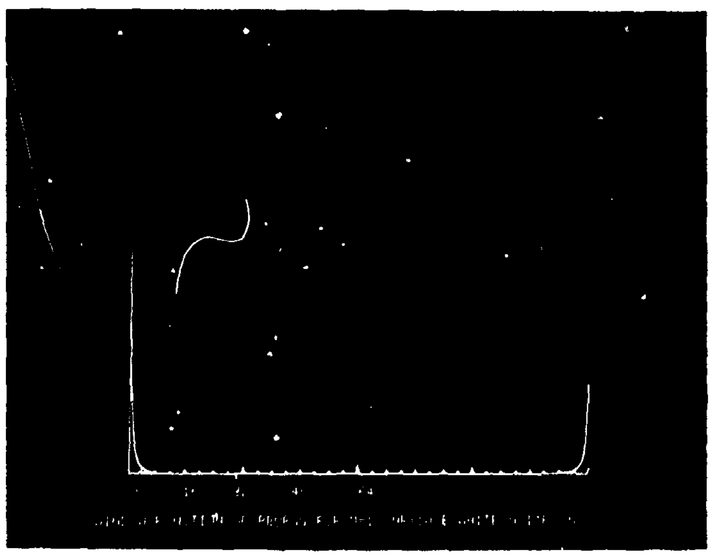


Fig. 15-b

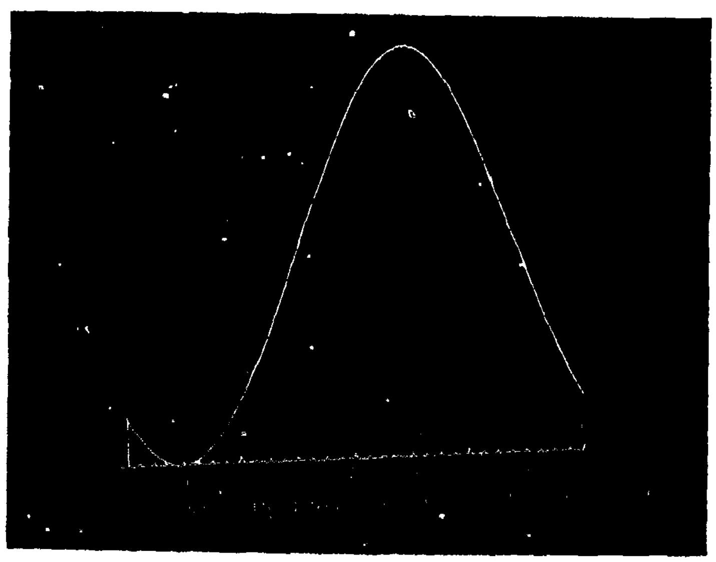


Fig.15-c

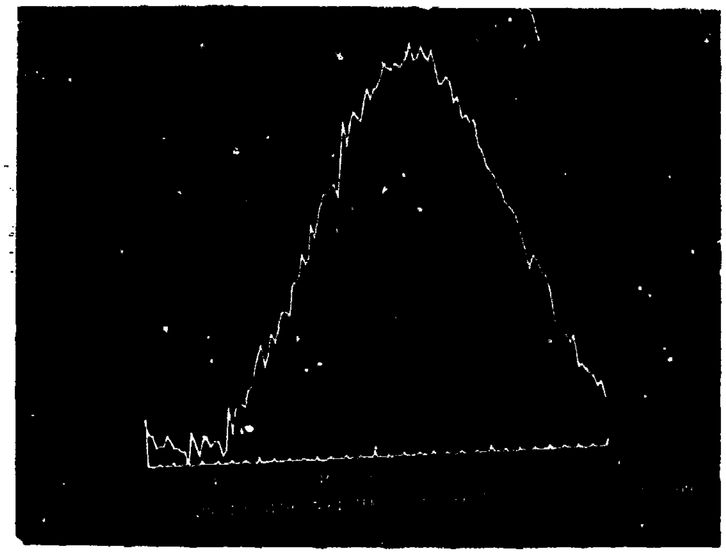


Fig.16-a

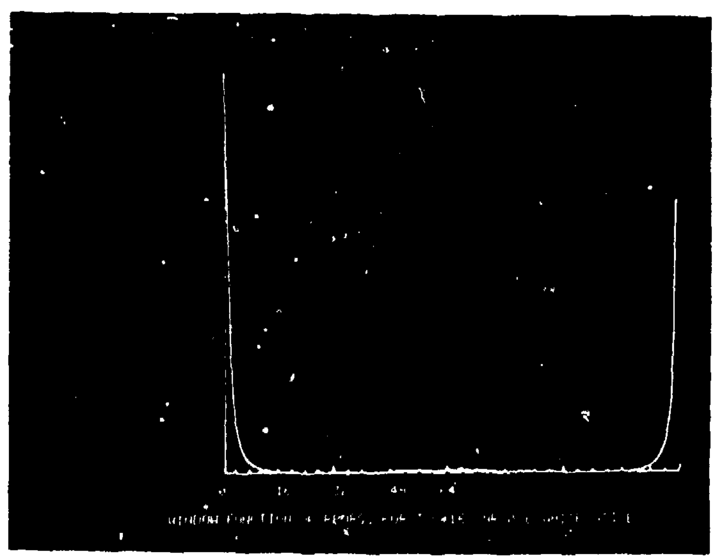


Fig.16-b

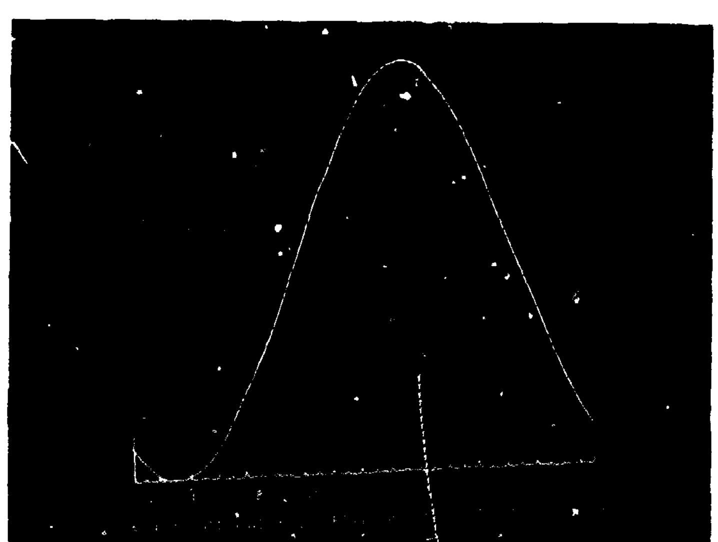


Fig.16-c

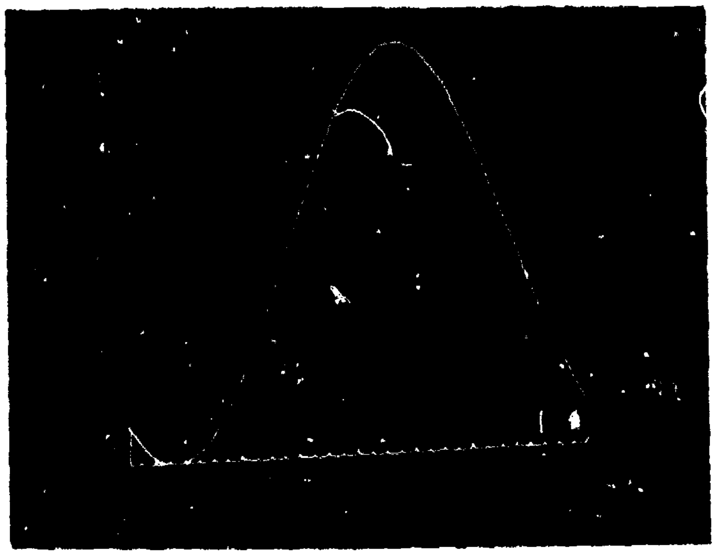


Fig. 17-a

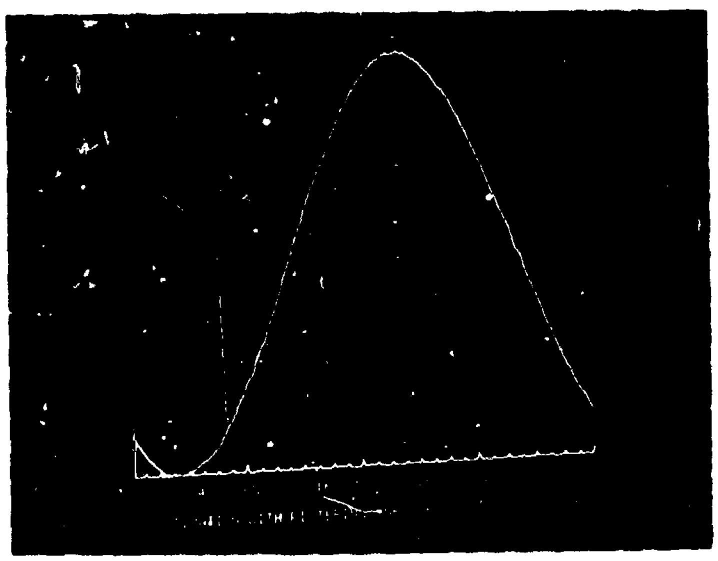
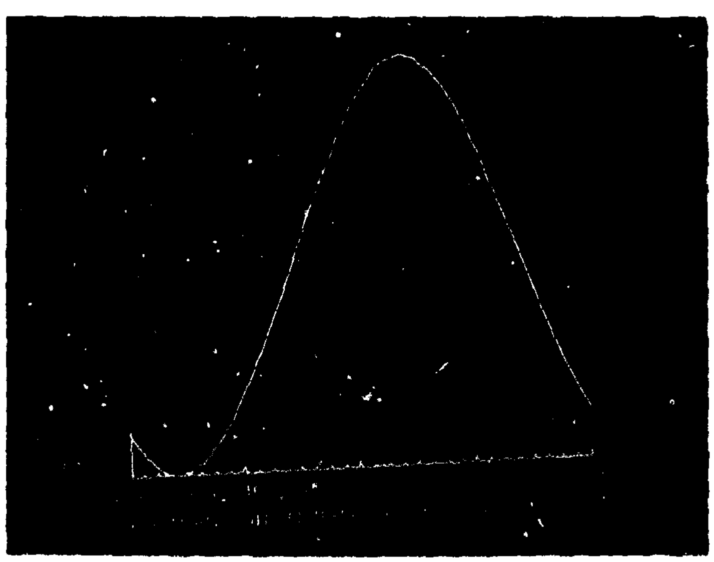


Fig.17-b



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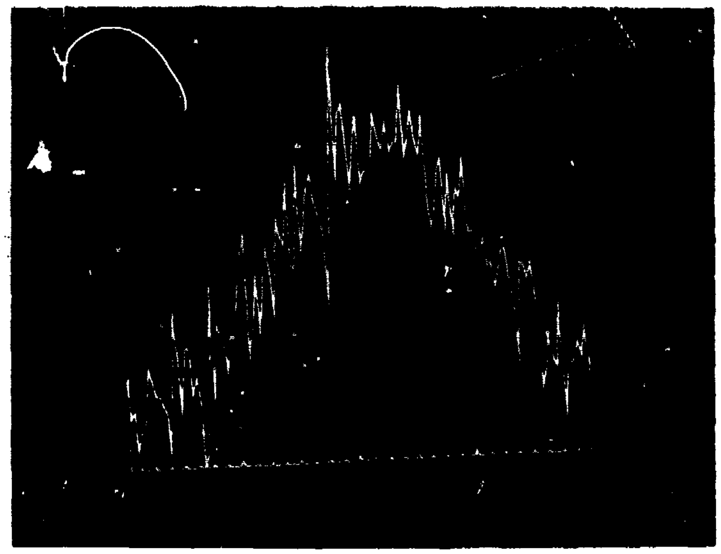
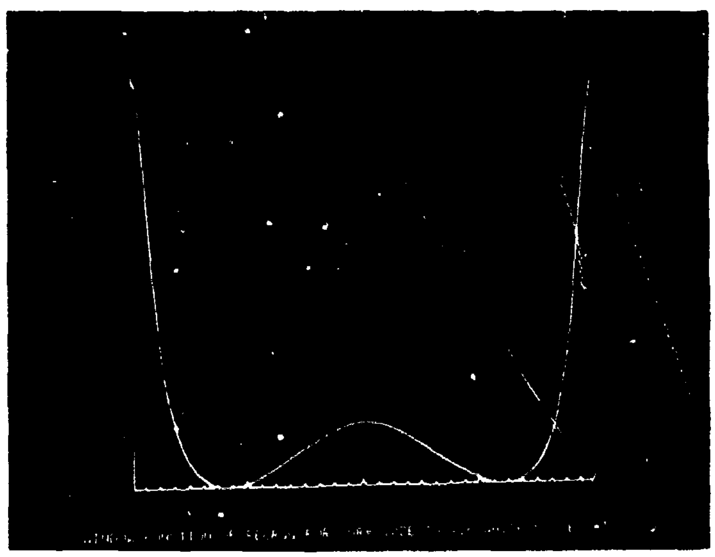


Fig. 18-a



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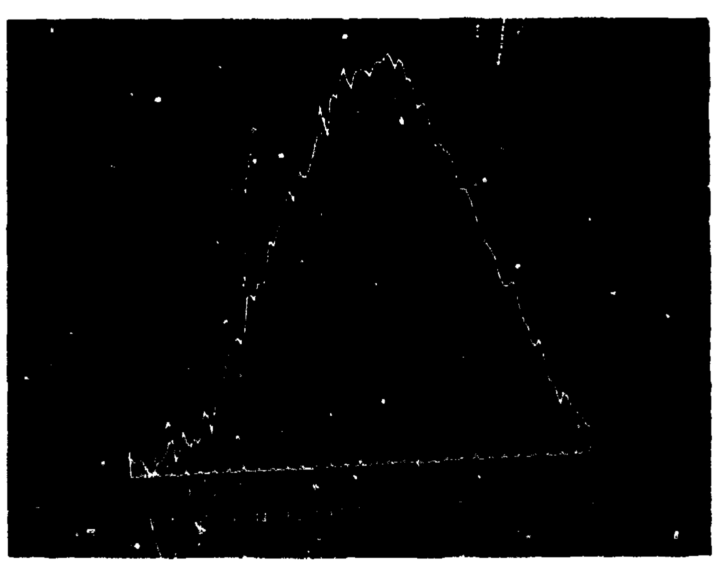

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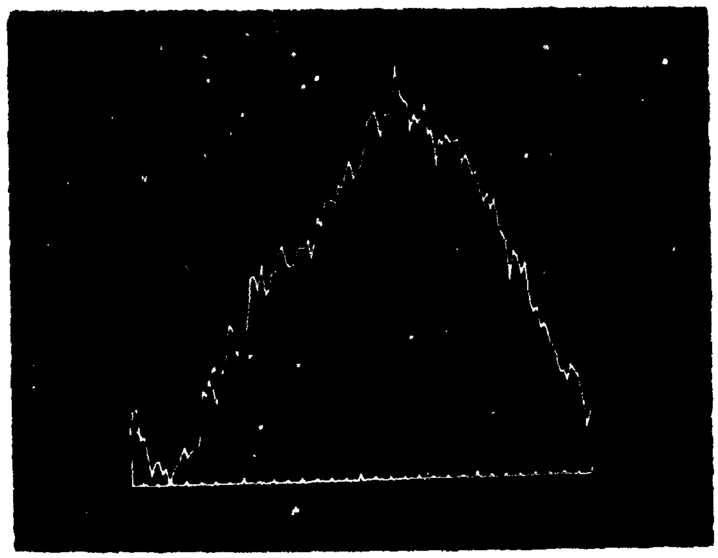
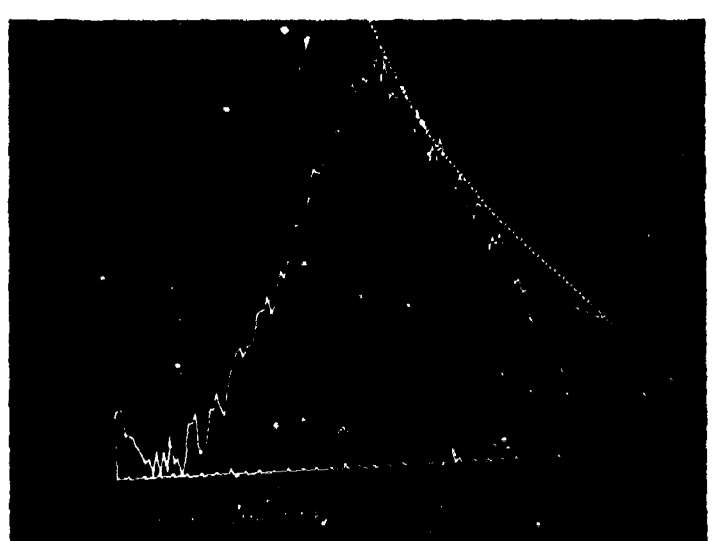


Fig. 19-a



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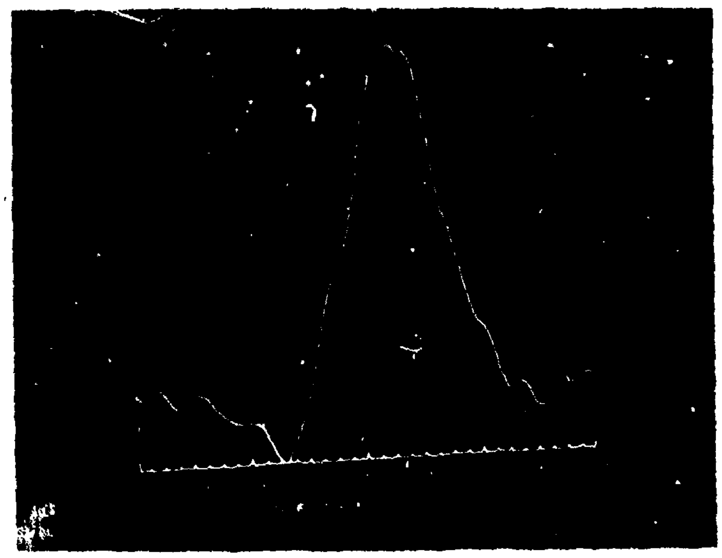


Fig.20-a

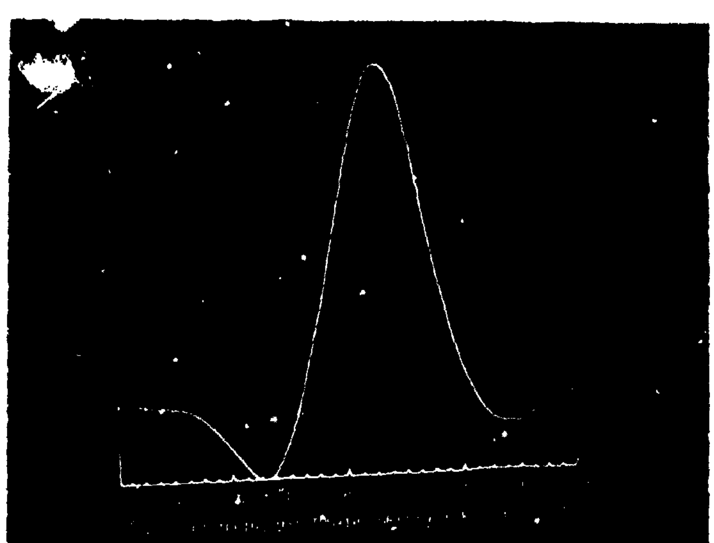


Fig.20-b

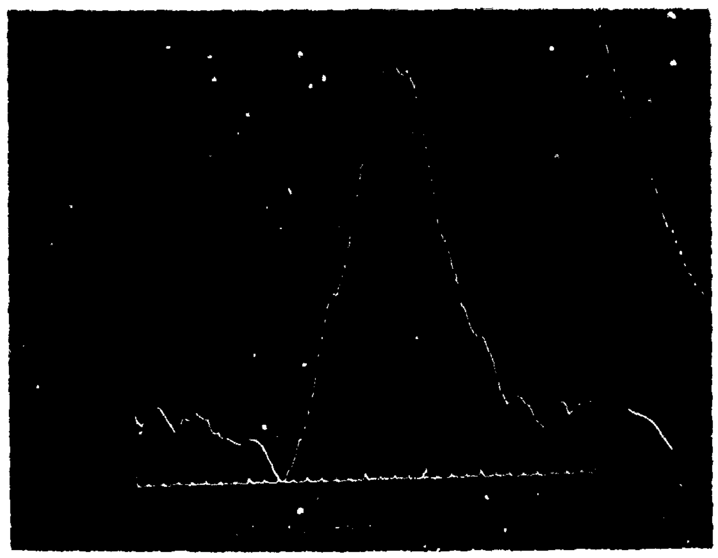


Fig.21-a

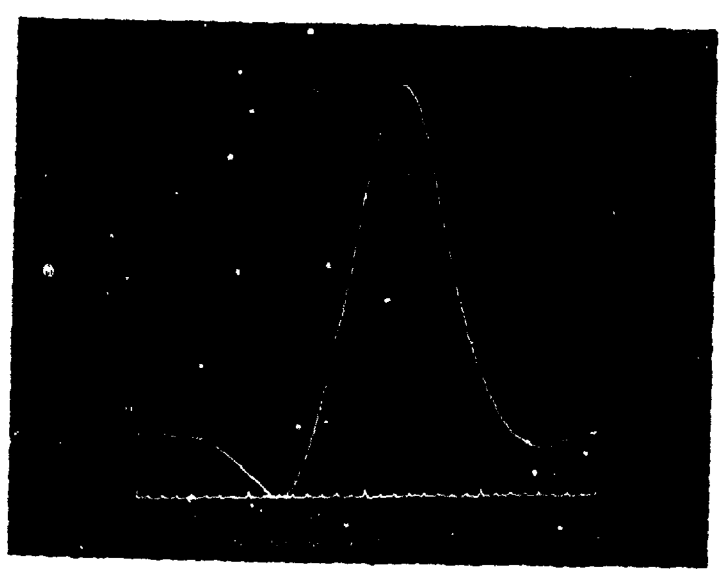


Fig. 21-b

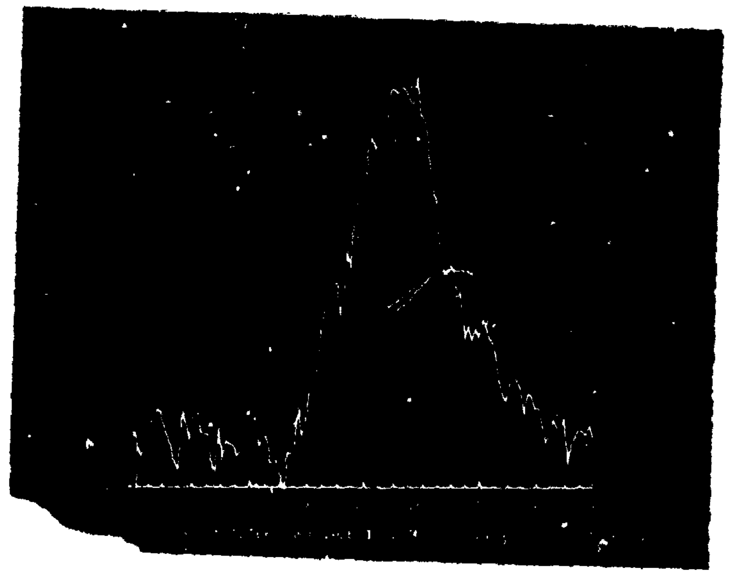
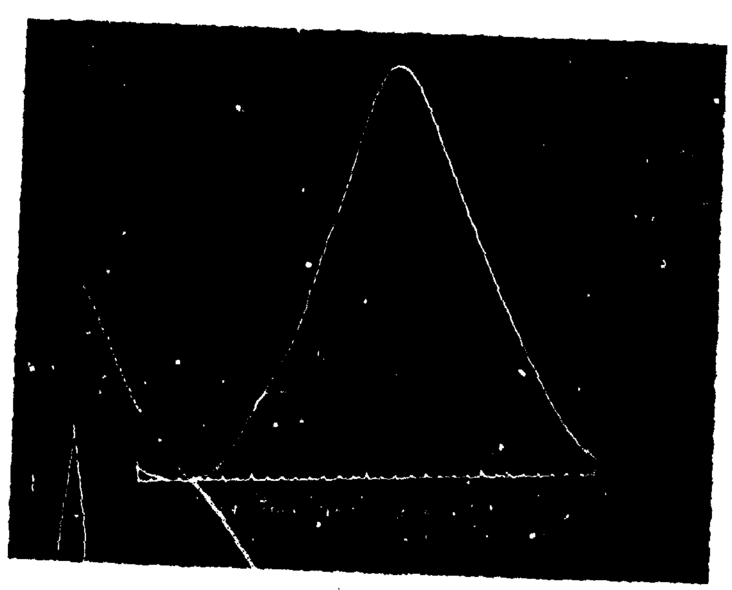


Fig.22-a



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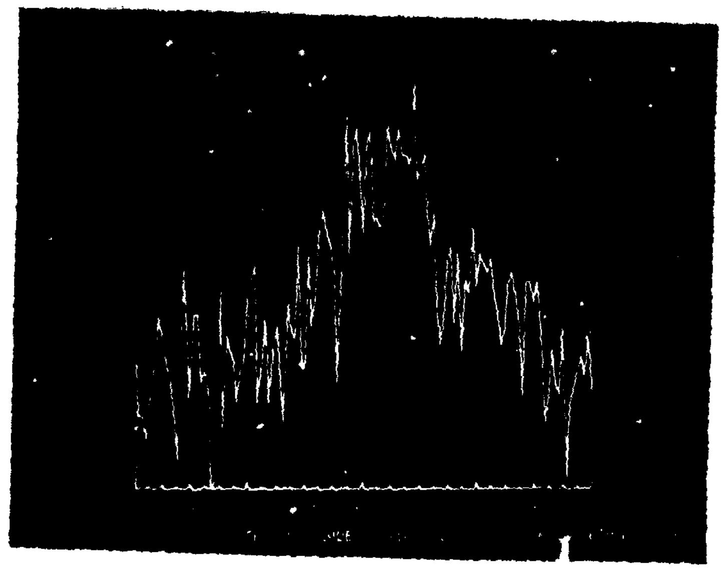

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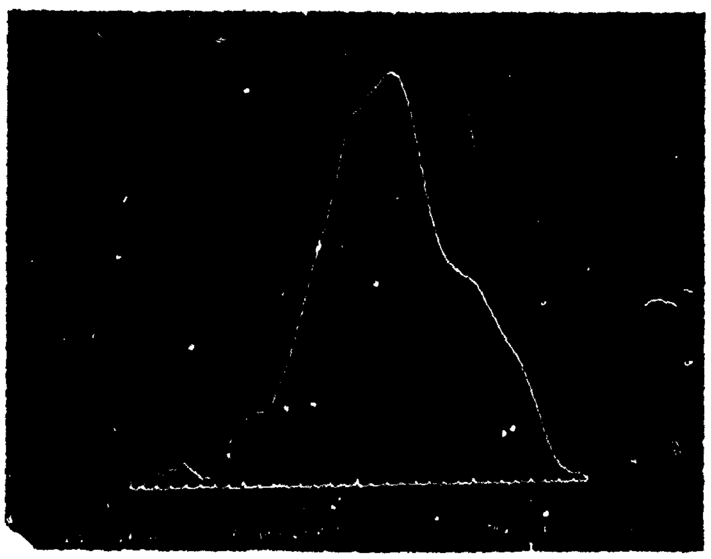
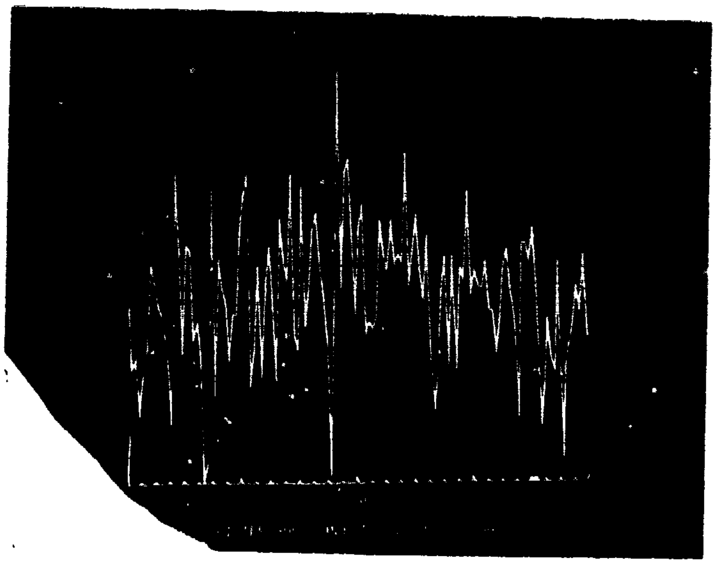


Fig.23-b



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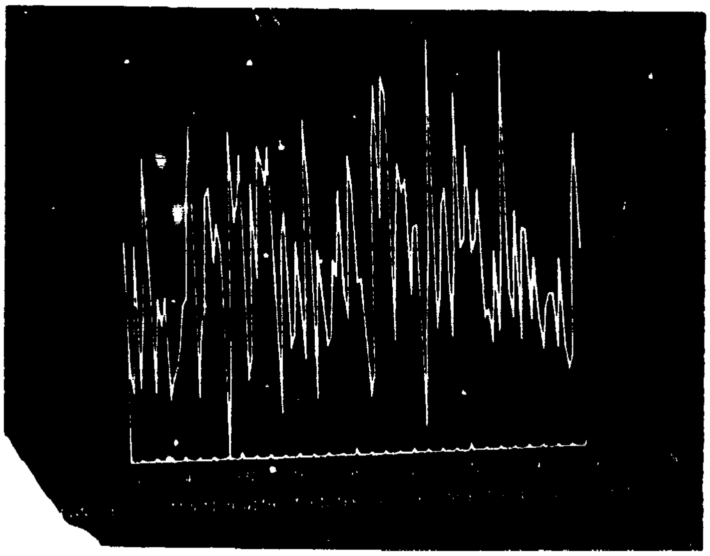

Fig. 24-b

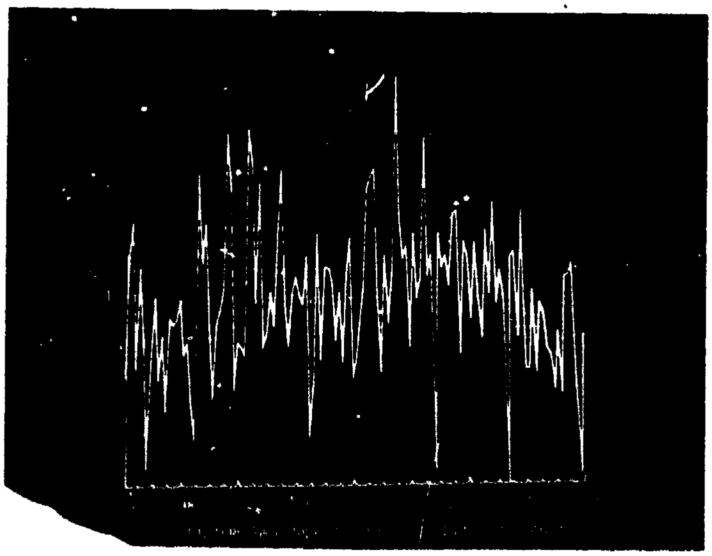

Fig. 24-c

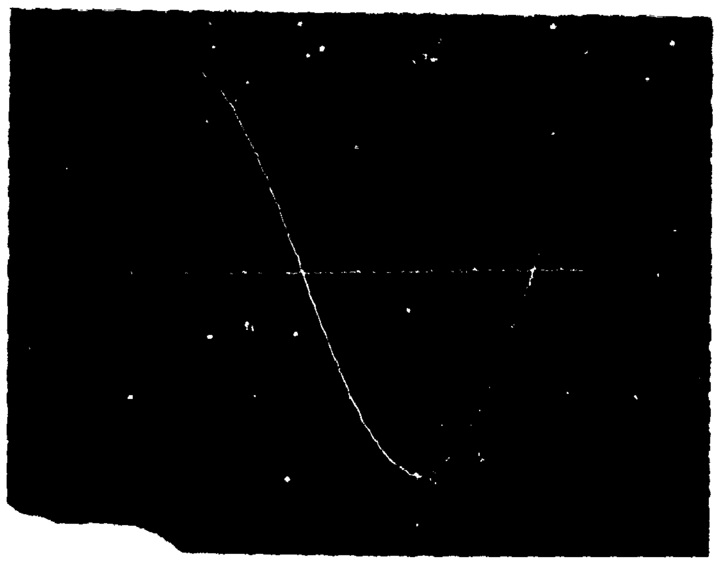

Fig.25-a

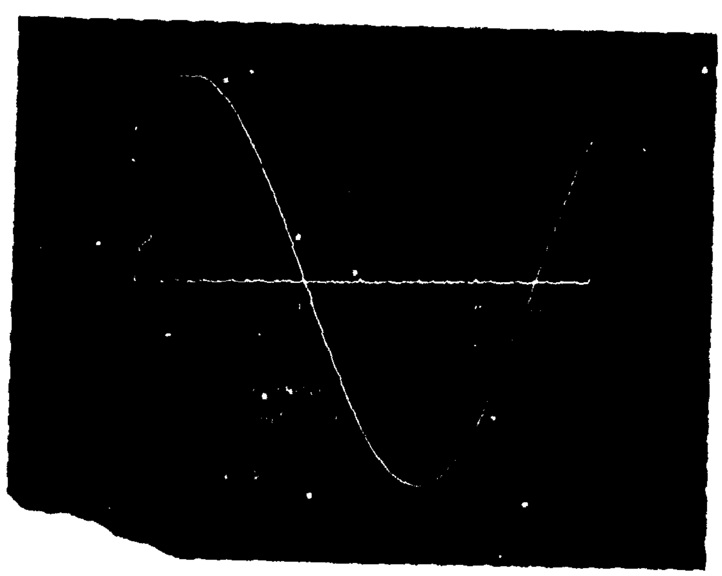

Fig.25-b

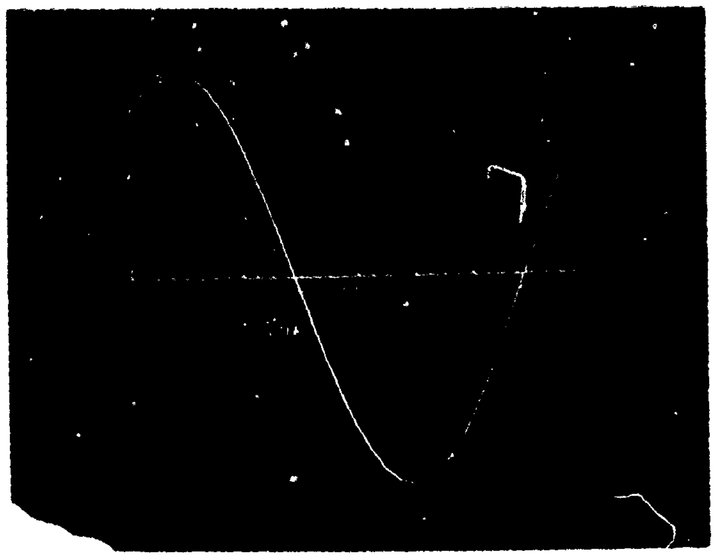

Fig.26-a

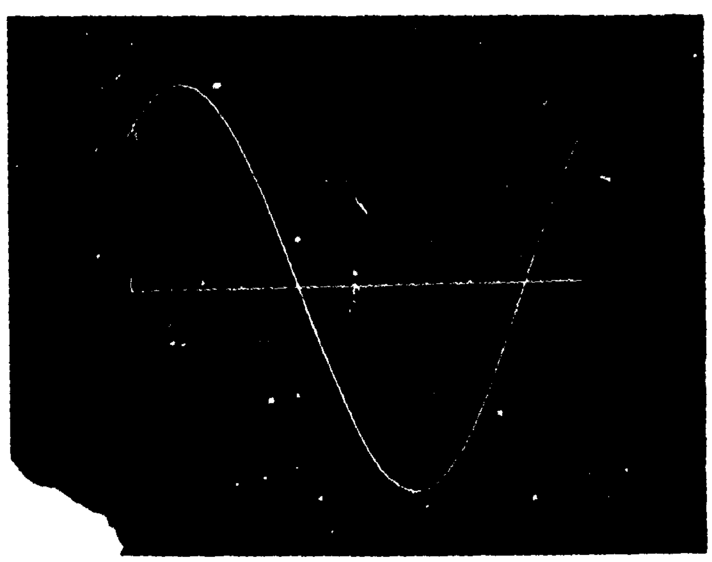

Fig.26-b

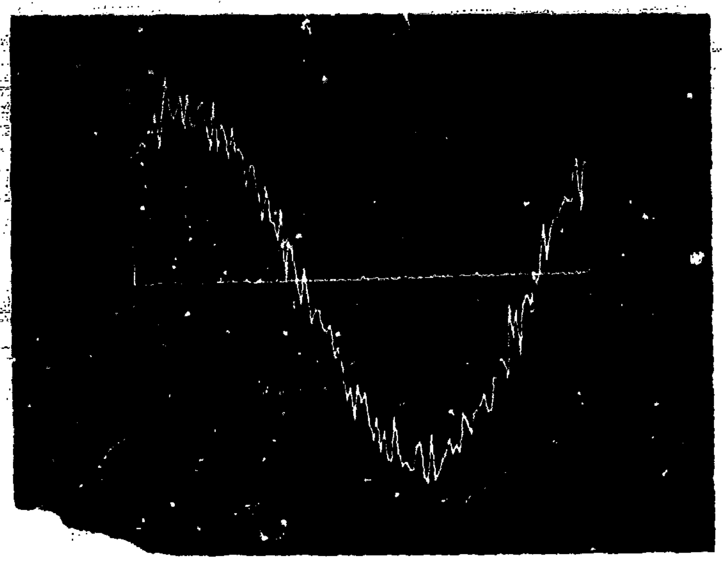

Fig.27-b

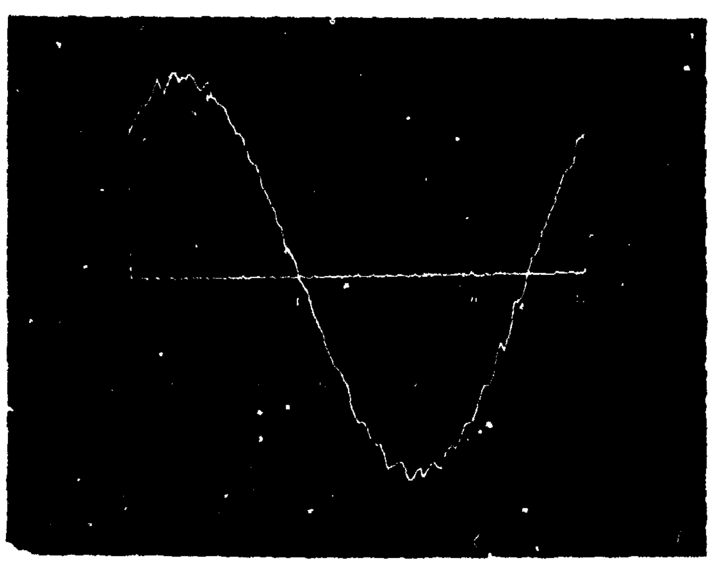

Fig.27-a

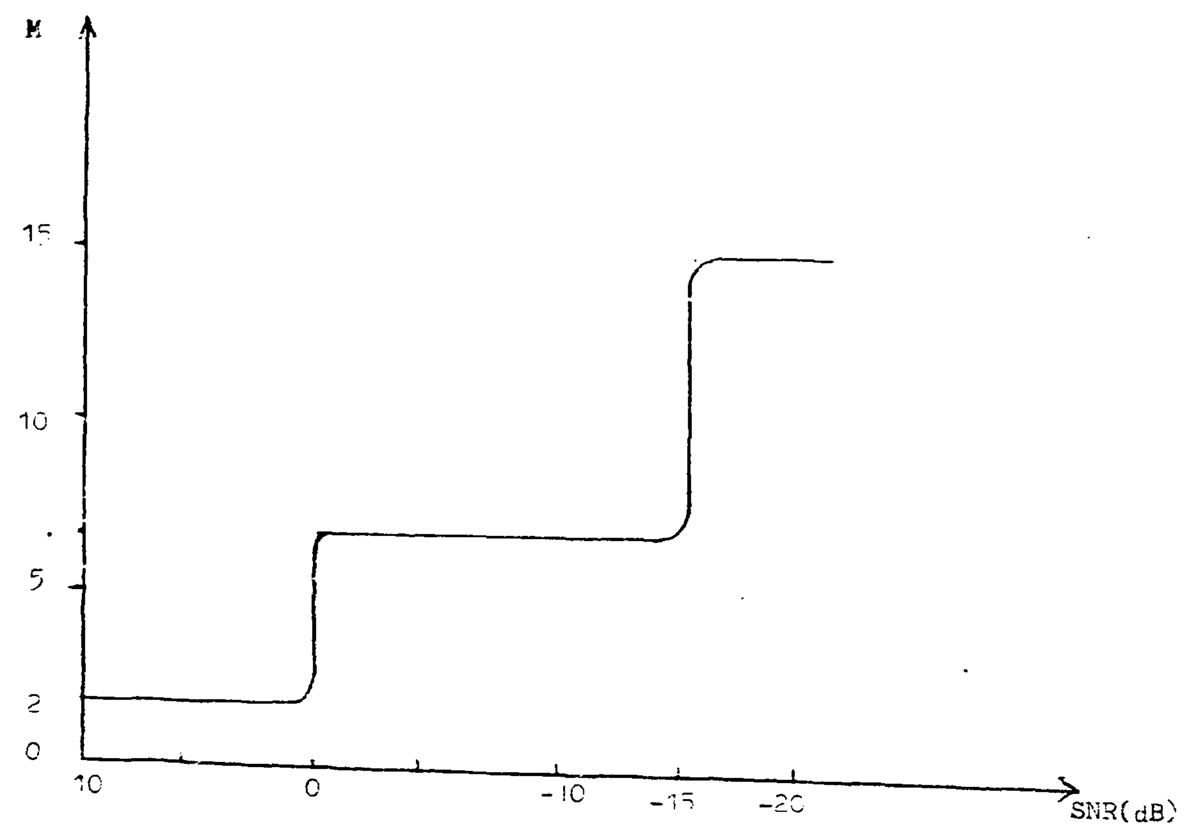

Fig. .28 The relationship of signal-to noise ratio(SNR) and segment M.

Reference A

(A) Burg's Recursive Algorithm

Let $X(t)$ denote a sample function of this process, which is sampled uniformly at the rate $1/T_s$ to produce a time series consisting of N samples, as shown by

$$\{X(n)\} = \{X(1), X(2), \ldots\ldots\ldots, X(N)\} \qquad (A1)$$

where $X(n)$ denotes the nth sample of the series.

The autocorrelation function of the time series $X(n)$ for a time lag of mT_s seconds and time index n is defined by

$$R_X(n,m) = E\,[X(n,m)\; X^*(n)] \qquad (A2)$$

where $E[\;]$ denotes the expectation, and the asterisk denotes the complex conjugate operation, With the random process $X(t)$ assumed wide-sense stationary, the autocorrelation function $R_X(n,m)$ dependent only on the time lag mT_s, as shown by

$$R_X(n,m) = R_X(m) \qquad (A3)$$

Also, the autocorrelation function exhibits conjugate symmetry, i., $R_X(m)$ is a Hermitian function

$$R_X(-m) = R_X^*(m) \qquad (A4)$$

Suppose that the time series $\{X(n)\}$ is applied to a linear digital filter of impulse response $\{h(n)\}$ of order P. The filter produces an output time series $\{y(n)\}$ that is designed to approximate a desired time series $\{d(n)\}$ as shown in Figure A1. and the resultant error time series is

$$e(n) = d(n) - y(n) \qquad (A5)$$

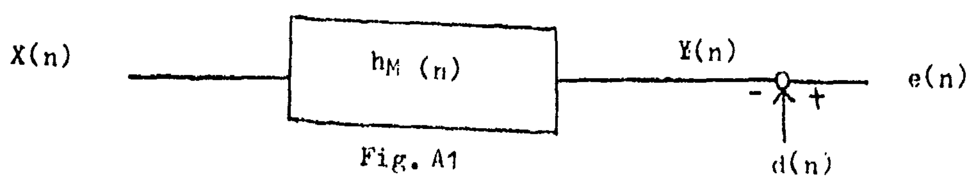

Fig. A1

For a special case of a filter designed to predict the value of a random process X(t), one time unit ahead by using the present and past values of the time series X(n), i.e.

$$d(n)=X(n+1)$$

A prediction-error filter is introduced. Fig. A2 shows the functional relationship between the linear prediction filter, characterized by the impulse response $\{h(n)\}$, and the prediction filter, characterized by the impilse response $\{w(n)\}$.

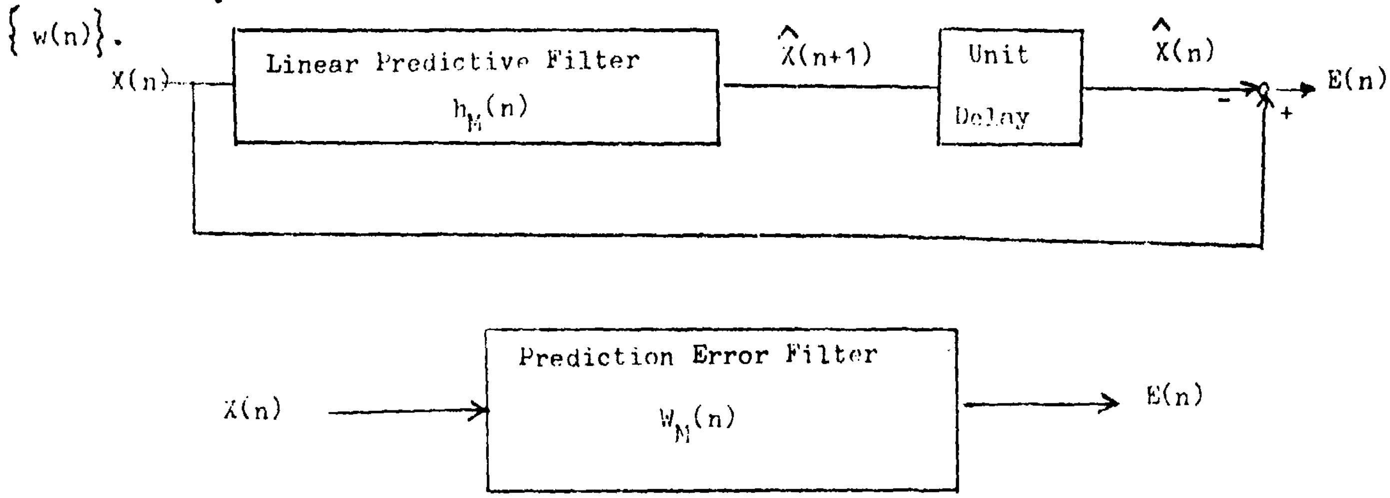

Fig.A2 : Illustrating the relationship between the predictive filter and prediction-error filter

The prediction-error filter equation expressed in matrix form is given by [12]

$$\begin{bmatrix} R_X(0) & R_X(-1) & \cdots & R_X(1-M) \\ R_X(1) & R_X(0) & \cdots & R_X(2-M) \\ \vdots & \vdots & & \vdots \\ R_X(M-1) & R_X(m-2) & \cdots & R_X(0) \end{bmatrix} \begin{bmatrix} 1 \\ W_M(1) \\ \vdots \\ W_M(M) \end{bmatrix} = \begin{bmatrix} P_M \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (A6)$$

where

$$P_M = \sum_{K=0}^{M-1} W_M(K)\ R_X(-K)$$

is the prediction-error power

A recursive algorithm developed by Burg [13] for solving the set of prediction-error filter equation is availible.

Suppose that we know the solution to the set of p equations pertaining to a prediction filter of order p, as shown by (A6). We can take the complex conjugate of both sides of (A6) and recognize that

(1) The prediction-error power P_M is a real quantity

(2) The autocorrelation matrix is Hermitian, i.e.

$$R_X^*(m) = R_X(-m)$$

We thus obtain

$$\begin{bmatrix} R_X(0) & R_X(1) & \cdots\cdots & R_X(M-1) \\ R_X(-1) & R_X(0) & \cdots\cdots & R_X(M-2) \\ & & \vdots & \\ R_X(1-M) & R_X(2-M) & \cdots\cdots & R_X(0) \end{bmatrix} \begin{bmatrix} 1 \\ W^*_{M-1}(1) \\ \vdots \\ W_{M-1}(M-1) \end{bmatrix} = \begin{bmatrix} P_M \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (A7)$$

(A7) can be rearranged as

$$\begin{bmatrix} R_X(0) & R_X(-1) & \cdots\cdots & R_X(1-M) \\ R_X(1) & R_X(0) & \cdots\cdots & R_X(2-M) \\ & & \vdots & \\ R_X(M-1) & R_X(M-2) & \cdots\cdots & R_X(0) \end{bmatrix} \begin{bmatrix} W^*_{M-1}(M-1) \\ W^*_{M-1}(M-2) \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ P_M \end{bmatrix} \qquad (A8)$$

(A6) pertains to a prediction-error filter of order p operated $(p=M)$

in the forward direction. On the other hand, (A8) pertains to a prediction-error filter of the same order except that is operated in the backward direction. This is illustrated in Fig. A3

$$\{X(1), X(2), \ldots\ldots, X(N)\} \longrightarrow \boxed{\{1, w_M(1), \ldots\ldots\ldots, w_M(M)\}} \longrightarrow O/P$$

(a)

$$\{X(1), X(2), \ldots\ldots, X(N)\} \longrightarrow \boxed{\{w_M^*(M), w_M^*(M-1), \ldots\ldots, 1\}} \longrightarrow O/P$$

(b)

Fig.A3: Illustrating : (a) forward and (b) backward prediction error filtering

Combining (A6) and (A8) to expand the number of prediction error filter equations by one is as follows:

$$\begin{bmatrix} R_X(0) & R_X(-1) \ldots\ldots & R_X(1-M) & R_X(-M) \\ R_X(1) & R_X(0) \ldots\ldots & R_X(2-M) & R_X(1-M) \\ & & & \\ R_X(M-1) & R_X(M-2) \ldots\ldots & R_X(0) & R_X(-1) \\ R_X(M) & R_X(M-1) \ldots\ldots & R_X(1) & R_X(0) \end{bmatrix} \left[\begin{bmatrix} 1 \\ w_{M-1}(1) \\ \cdot \\ \cdot \\ \cdot \\ w_{M-1}(M-1) \\ 0 \end{bmatrix} + w_M \begin{bmatrix} 0 \\ w^*_{M-1}(1) \\ \cdot \\ \cdot \\ \cdot \\ w^*_{M-1}(1) \\ 1 \end{bmatrix} \right]$$

$$= \left[\begin{bmatrix} P_M \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \Delta_M \end{bmatrix} + w_M(M) \begin{bmatrix} \dot{M} \\ 0 \\ \cdot \\ \cdot \\ \cdot\cdot \\ \cdot \\ P_M \end{bmatrix} \right] \qquad \text{(A9)}$$

However, for the corresponding prediction-error filter of order (m+1) we have

$$\begin{bmatrix} R_X(0) & R_X(-1) & \cdots & R_X(1-M) & R_X(-M) \\ R_X(1) & R_X(0) & \cdots & R_X(2-M) & R_X(1-M) \\ & & \vdots & & \\ R_X(M-1) & R_X(M-2) & \cdots & R_X(0) & R_X(-1) \\ R_X(M) & R_X(M-1) & \cdots & R_X(1) & R_X(0) \end{bmatrix} \begin{bmatrix} 1 \\ W_M(1) \\ \vdots \\ W_M(M-1) \\ W_M(M) \end{bmatrix} = \begin{bmatrix} P_{M+1} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \qquad (A10)$$

Comparing with (A9) and (A10), we deduce that

$$W_M(K) = W_{M-1}(K) + W_M(M)\, W_{M-1}(M-K) \quad ; K=0,1,\ldots,p(=M) \qquad (A11)$$

$$P_{M+1} = P_M + W_M(M)\Delta^*_M \qquad (A12)$$

and

$$0 = \Delta^*_M + W_M(M)P_M \qquad (A13)$$

In the recursive formula of (A11), note that for all values of M

$$W_M(K) = \begin{cases} 1 & \text{for } K=0 \\ 0 & \text{for } K>M \end{cases}$$

(A13) will always has a solution provided that $P_M > 0$, thus using (A13) to eliminate Δ^*_M from (A12), we get

$$P_{M+1} = P_M\left[\,1 - |W_M(M)|^2\,\right]$$

By analogy with the transmission of power through a terminated two-port network, we may view $W_M(M)$ as a " reflection coefficient".

COMPUTATIONAL PROCEDURE

Summarizing the steps in the recursive procedure involved in calculating the prediction-error filter coefficients and related values of the spectral density and autocorrelation function, they are

(1) For thegiven time series $X(n)$, $n=1,2,\ldots\ldots N$ and $M=0$.

Calculate the power

$$P_1=\frac{1}{N}\sum_{n=1}^{N} X(n)\; X^*(n)$$

and calculate the reflection coefficient and the prediction-error power by

$$W_1=\frac{-2\sum_{n=1}^{N-1} X^*(n)\; X(n+1)}{\sum_{n=1}^{N-1}\left[\,|X(n)|^2 + |X(n+1)|^2\,\right]}$$

$$P_2= P_1\left[\,1-|W_1(1)|^2\,\right]$$

(2) Increament M by 1, and calculate the next values:

$$P_M(n)= P_{M-1}(n) + W_M^*(M)\; q_{M-1}(n)$$

$$q_M(n)=q_{M-1}(n+1) + W_M(M)\; P_{M-1}(n+1)$$

where

$$P_0(n)= X(n)$$

$$q_0(n)= X(n+1)$$

(3) Calculate the refle on coefficient

$$W_M(M)=\frac{-2\sum_{n=1}^{N-M} P^*_{M-1}(n)\; q_{M-1}(n)}{\sum_{n=1}^{N-M}\left[\,|P_{M-1}(n)|^2 + |q_{M-1}(n)|^2\,\right]}$$

and the corresponding value of the error prediction power

$$P_{M+1} = P_M \left[1 - |W_M(M)|^2 \right]$$

(4) Calculate the remaining coefficients of the prediction error filter by

$$W_M(K) = W_{M-1}(K) + W_M(M)\, W^*_{M-1}(M-K) \quad ; \; K=1,2,\ldots,M-1$$

(5) Repeat steps (2) to (4) for each value of M up to the optimum value p(=M) for which the final prediction-error

$$(FPE)_M = \frac{(N+M+1)}{(N-M-1)} P_{M+1}$$

is a minimum.

(6) We now have all the quantities required for calcuation the extrapolated value of the autocorrelation function.

$$R_X(M) = -W_M(M)\, P_M - \sum_{K=1}^{M-1} W_{M-1}(K)\, R_X(M-K)$$

and for calculating the spectral density estimate

$$\hat{S}_X(f) = \frac{P_{M+1}}{2B \left| 1 + \sum_{K=1}^{M} W_M(K) \exp(-2\pi k f T_s) \right|^2}$$

where T_s is the sampling period and X(t) is the band-limited to -B,B.

When the process X(t) is sampled at the Nyquist rate, we have T=1/2B

The computation is thereby completed.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Time delay estimation using Hassab-Boucher windows is extensively examined by computer simulation for both known and unknown (random) signals. Optimum number of data segments is determined for the best result in time delay estimate. For the unknown signal case, the Burg's maximum entropy spectral estimation algorithm is used to estimate the signal spectral density used in the design of Hassab-Boucher windows.

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